DYNAMIC ASPECTS OF FEED MECHANISM

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ABSTRACT

Present paper is continuing several previous works concerning determination of constructive parameters of feed mechanism for die sinking electro discharge machines.

KEYWORDS: electro discharge machine, feed, electro-mechanical feed

1. INTRODUCTION

Feed mechanism conceived, designed and materialized inside IESP Department of Transilvania University of Brasov, is composed from a low inertia driving motor, a worm reducer and a screw-nut ensemble.
Kinematical, this mechanism is presented in figure 1.

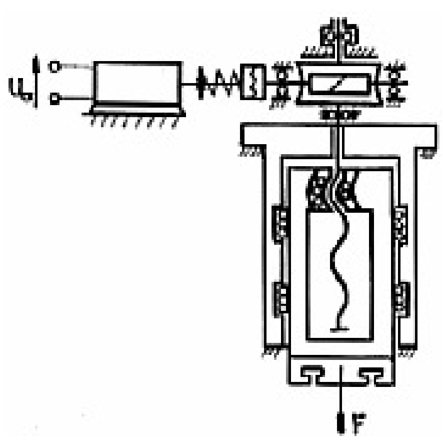

Fig. 1 Kinematical design of feed mechanism

2. DRIVING MOTOR MODELLING

Taking in consideration that the feed mechanism of electro discharge machining are working at severe dynamic regime, for driving such system a low inertia DC motor was chosen. This motor has practically linear characteristics, a high starting torque and a low inertia moment.
Criteria in choosing this drive were a good switching in each working regime, it has no iron armature and this fact has a good influence on linear dependence of current and inductive electromotive voltage. Even at low speed, this drive can endure very well overloads. This is possible because of its high admissible current densities in armature which can reach up to 100 A/mm in short stages and up to 45 A/mm for longest stages.
Amongst low inertia DC motors, the ones with disk armature are wide spreading. The main components are: fixing flanges, disc armature and collector-commutation brush system.
Lateral flanges are in fact support for permanent magnets which create axial magnetic field in which is spinning disc armature. The shape of these magnets is cylindrical; they are independent and fixed on steel fixture.
The magnets arrangement is creating a magnetic flux and magnetic induction in air gap of motor is satisfying, from reached values, the prerequisite of electromechanical feed mechanism. Non iron rotor armature is made from glass fibre impregnate with epoxy resin.
Armature winding is made from cooper sheets, obtained by stamping in 2, 3, 6 or 8 serial layers. The advantages of such practical solution are short ending windings and higher electromotive voltage.

a. Disk rotor electromagnetic couple

Electromagnetic torque determination for this type of motor is based on generalised model for electrical machines. In this case, considering permanent excitation is represented only one pair of brushes on q axis and a number of 2a current circuit (fig 2).

On d axis are placed excitation poles, physically made of permanent magnets. For mathematical modelling purposes, permanent magnets are assimilated with two windings crossed by an equivalent I_{Mf} amperage. In this way, total magnetic flux on one pole is replaced by fictive amperage, which is proportional with an electromotive voltage matching motor magnetic circuit.

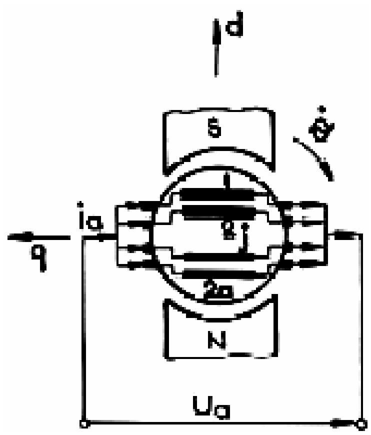

Fig. 2 Equivalent circuit for a low inertia motor

According with general theory of electrical machines and considering the schematic of an electrical motor with permanent excitation, the expression of electromagnetic couple is:

$$m_e = \sum_{i=1}^{2a} b\frac{z}{4a}\cdot\rho_d\cdot K_q\cdot i_{qi}\ \text{[Nm]} \qquad (1)$$

where:
$z\cdot\rho_d / 4a$, is excitation magnetic flux related to amperage path,
b, pairs number of electrical machine,
$K_g = 2/\pi$ - distribution factor for rotor winding,
z – winding number of rotor armature.
Expression of electromagnetic couple is:

$$m_e = K_m\cdot i_Q \qquad (2)$$

where:
$K_m = b\cdot z\cdot\rho_d / 2\pi a$ couple constant of electrical machine
ρ_d – excitation flux
$i_Q = 2a\cdot i_Q$ – real amperage at brushes.

b. Voltage equation in electrical motor
In equivalent circuit presented in figure 2, every amperage path is considered to have its own inductivity L_q, a mutual inductivity L'_q versus every amperage path and a dispersion inductivity L_{qs}.
In case of a command in armature, excitation voltage is constant and total magnetic excitation flux is:

$$\psi_d = L_{Md}\cdot I_{Mf} \qquad (3)$$

where:
L_{Md} is corresponding inductivity of circuit.
In this way, the equation of electrical voltage for d axis amperage path characterise an amperage path for the model presented:

$$-b\cdot\omega\cdot K_q\cdot\frac{z}{4a}\cdot\rho_d - L_q\cdot\frac{d_{iq}}{dt} - (2a-1)\cdot L'_q\frac{d_{iq}}{dt} = R_q\cdot i_q - u_q \ldots\ldots[4] \qquad (4)$$

For 2a amperage paths, the voltage equation for motors with permanent excitation is:

$$u_Q = -u_e + R_Q\cdot i_Q + L_Q\cdot\frac{di_A}{dt} \qquad (5)$$

In equation 5 the following notation where used:
$R_Q = \frac{R_q}{2a}$ armature electrical resistance measured at brushes,
$u_e = -k_e\cdot\omega$ - electromotive voltage,
$u_Q = u_0$ – voltage at electrical motor terminals,
$L_Q = \frac{L_q + (2a-1)L'_q}{2a}$ - armature inductivity,
$i_Q = 2a\cdot i_q$ - real amperage at brushes.
Considering that resistant couple at endless screw is Mm, the couple developed by the electrical motor should be greater. This last couple should be even greater because it has to overpass the motor frictional static couple (M_f), viscous friction couple (M_{fv}) and also acceleration couple $Id\omega/dt$ which include motor inertia couple, load inertia couple and all inertia moment of transmission, reported to motor shaft:

$$m_e - M_m - M_f - M_{fv} = J\frac{d\omega}{dt} \qquad (6)$$

In conditions mentioned above, in stationary regime, couples and voltages equations are:

$$u_Q = R_Q\cdot i_Q + K_e, \qquad (7)$$

$K_m \cdot i_Q = M_m + M_f$ (8)

where:

$K_e = \frac{b \cdot z}{2\pi \cdot a} \cdot \psi_p$ is motor voltage constant

By eliminating amperage from relations (7) and (8) the result is:

$$u_Q = \frac{(M_m + M_f) \cdot R_Q}{K_m} + K_e \cdot \omega \quad (9)$$

In real conditions with no load and considering the real frictions, motor angular speed ω_0 is:

$$\omega_0 = \frac{u_Q}{K_e} - \frac{M_f \cdot R_Q}{K_e \cdot K_m} \quad (10)$$

The relationship between angular speed and couple is described by the mechanical characteristic equation:

$$\omega = -R_m \cdot M_m + \frac{u_Q}{K_e} - R_m \cdot M_f \quad (11)$$

where:

$R_m = \frac{R_Q}{K_e \cdot K_m}$ is mechanical characteristic slope or control revolution constant.

c. Low inertia motor parameters

Being a non iron rotor, the motor selected is characterised by a large air gap due to absence of iron and armature notches. These constructive particularities are reducing the influence of armature reaction over the behaviour of permanent magnets and are determining, in practice, a symmetrical induction distribution.

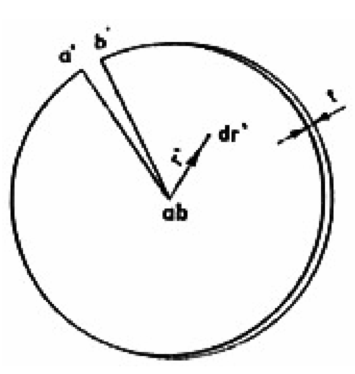

Fig. 3 Armature model

Armature is approximate with a disk shape conducting surface with r radius, t_g thickness and a ρ_d resistivity according with the model presented in figure 3.

Disk resistance from a-a to b-b, which are equipotential lines, is obtained from following relation:

$$R_Q = \frac{(1{,}2-1{,}3)\rho_d \cdot l}{S} = \frac{(1{,}2-1{,}3)2\pi(r/2)\rho_d}{r \cdot t_g} = \frac{(1{,}2-1{,}3)\pi\rho_d}{t} \text{...[12]} \quad (12)$$

where l is the length of conducting surface, S area and the average length of surface is half of radius and represent average length of the same.

As a remark, disk electrical resistance is independent towards radius, being dependent only with disk thickness.

Armature inductivity can be ascertained with the following relation:

$$L_Q = \frac{L_q + (2a-1) \cdot L_q'}{2a} \quad (13)$$

and motor constant K_M with relation:

$$K_M = \int_0^r \alpha_p \frac{B_r}{K_\delta} r' \cdot dr' = \frac{1}{2} \alpha_p \frac{B_r}{K_\delta} r^2 \quad (14)$$

where:

α_p is the disk portion under a pole,

B_r is induction after radius,

K_δ is magnetic flux dispersion factor.

It can be observed that motor constant K_M for a disk rotor is proportional with radius square. Therefore the weight and the price for an electrical motor disk rotor, at the same parameters, at lowest than an electrical motor with cylindrical armature.

Electromagnetic couple will result from replacing supply amperage i_a defined by voltage equation:

$$M = K_M \cdot i_Q = \frac{1}{2} \cdot \alpha_p \cdot \frac{B_r}{K_\delta} \cdot r^2 \cdot \frac{U_a - \frac{1}{2} \cdot \alpha_p \cdot \frac{B_r}{K_\delta} \cdot r^2 \cdot \omega}{(1{,}2-1{,}3) \cdot \rho \cdot \frac{\pi}{1}} \quad (15)$$

Motor couple constant K_M and K_e constant where determined with relation:

$$K_M = K_e = \frac{pN}{2\pi a} \cdot \psi_p = 0{,}06[N \cdot m / A]; \left[\frac{V}{rad \cdot s^{-1}}\right] \quad (16)$$

For M = 0, no-load motor angular speed is:

$$\omega_0 = \frac{2u_Q \cdot K_\delta}{\alpha_p \cdot B_r \cdot r^2}, \qquad (17)$$

and for ω = 0, starting couple is:

$$M_p = \frac{1}{2(1{,}2-1{,}3)\rho \cdot \pi} \cdot \alpha_p \cdot \frac{B_r}{K_\delta} \cdot r^2 \cdot t \cdot u_a \qquad (18)$$

Rotor inertia moment was determined with the help of following relation:

$$J_d = \frac{K_j \cdot \gamma_a \cdot \pi \cdot t}{2} \cdot r^4 = 3{,}5 \cdot 10^{-4} [kg \cdot m^2], \qquad (19)$$

where K_j is the ratio between armature inertia moment with and without winding and γ_a is armature medium density.

Disk armature frictional viscous coefficient was obtained from:

$$C_{fv} = \frac{M_p}{\omega_0} = \frac{\alpha_p^2 \cdot B_r^2 \cdot r^4 \cdot t}{4\pi \cdot \rho \cdot K_\delta^2} [N \cdot m / rad \cdot s^{-1}] \qquad (20)$$

Motor time electromagnetic constant was determined with the relation between motor inertia couple and "magnetic frictional viscous coefficient":

$$T_{em} = \frac{J_d}{C_{fv}} = \frac{K_j \cdot \gamma_a \cdot \rho \cdot K_\delta^2}{\alpha_p^2 \cdot B_r^2} = 25 \cdot 10^{-3} [s] \qquad (21)$$

3. FORCES AND COUPLES VS. MOTOR SHAFT

In this particular mechanism, electrical motor is driving electrode plate with a worm gear and ball screw-nut mechanism (figure 4).

Because feeding mechanism has electrode support as actuator, is necessary to report to driving shaft the couple resulted from moving m weight with v_z speed. This can be done based on conserving energy law. According to this, the power corresponding to report value is equal to initial power taking in consideration system losses.

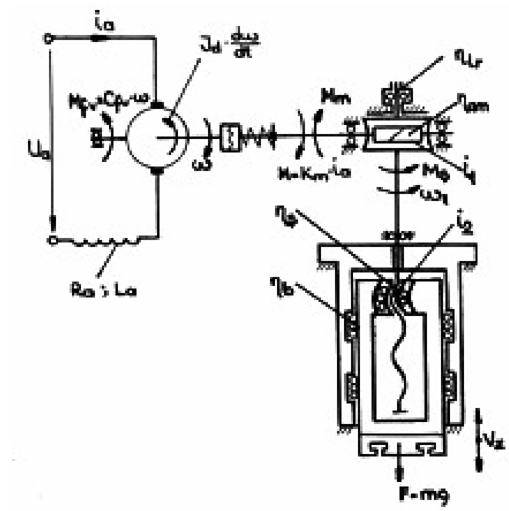

Fig. 4 Electromechanical feed mechanism

In order to obtain correct relations the following where taking in consideration: moments M_m and M_s at axes of endless screw and at driving shaft, angular speeds ω, ω_1, transmission ratios i_1, i_2 and overall efficiency η which is given by the following relation:

$$\eta = \eta_b \cdot \eta_s \cdot \eta_{lr} \cdot \eta_{am} \cdot \eta_{lm} \qquad (22)$$

where:

η_b – electrode guides efficiency (η_b = 0,97);
η_s – ball screw efficiency (η_s = 0.860);
η_{lr} – worm gear bearings efficiency (η_{lr} = 0,98);
η_{am} – gear bearings efficiency (η_{am} = 0,703);
η_{lm} – worm gear bearings efficiency (η_{lm} = 0,97).

Equalising powers at successive shafts, the following relations results:

$$M_S \cdot \omega_1 = \frac{1}{\eta_b \cdot \eta_s} \cdot m \cdot g \cdot v_Z ; M_S = \frac{1}{\eta_b \cdot \eta_s} \cdot m \cdot g \cdot \frac{v_Z}{\omega_1}$$
$$\Rightarrow M_S = \frac{1}{\eta_b \cdot \eta_s} \cdot m \cdot g \cdot \frac{1}{i_2} \qquad (23)$$

and

$$M_m \cdot \omega = \frac{M_S \cdot \omega_1}{\eta_{lr} \cdot \eta_{am} \cdot \eta_{lm}} ; M_m = \frac{M_S}{\eta_{lr} \cdot \eta_{am} \cdot \eta_{lm}} \cdot \frac{\omega_1}{\omega} \Rightarrow$$
$$M_m = \frac{1}{\eta} \cdot \frac{1}{i} \cdot m \cdot g \qquad (24)$$

where $i = i_1 \cdot i_2$

In order to obtain the reduced couple at motor shaft, at worm shaft moment was added dry frictional moment of the motor:

$$M_r = M_m + M_f \Rightarrow M_r = 4{,}91 \cdot 10^{-1} \text{ [Nm]} \qquad (25)$$

a. Mass and moments of inertia at motor shaft

Mass and moments of inertia at motor shaft was done it by replacing them with an equivalent inertia moment J_e, corresponding to a fictive part, which is having a kinetic energy equal to the sum of kinetic energy, stored by real elements. Where considered the inertia moments for the following elements: endless screw (J_m), worm gear and driving shaft (J_s) and m mass in translation movement:

$$J_e \cdot \frac{\omega^2}{2} = J_m \cdot \frac{\omega^2}{2} + J_S \cdot \frac{\omega_1^2}{2} + m \cdot \frac{v_Z^2}{2}, \qquad (26)$$

As resulted from previous equality equivalent inertia moment is:

$$J_e = J_m + J_S \cdot \frac{1}{i_1^2} + m \cdot \frac{1}{i^2}, [kg \cdot m^2] \qquad (27)$$

At equivalent inertia moment is added disk rotor moment of inertia:

$$J = J_e + J_d \ [kg \cdot m^2] \qquad (28)$$

b. Movement equation establishment for electromechanical feed mechanism

The dynamic of electromechanical feed mechanism will be defined by establishing the movement equation.

After mass, frictions and inertia moments were reduced to motor drive shaft, mechanism diagram can be represented, in a simplified version, as presented in figure 5.

Electrical motor is developing a moment M in order to overcame the resistant moment Mr from feed mechanism added to viscous frictional moments from electrical motor M_{fv}. If sum of two resistant moments is equal to the torque developed by the motor, then feed mechanism is working with constant speed, in other words in stationary regime.

The following relation

$$M - M_r - M_{fv} = 0 \qquad (29)$$

is representing the movement equation for the situation described above.

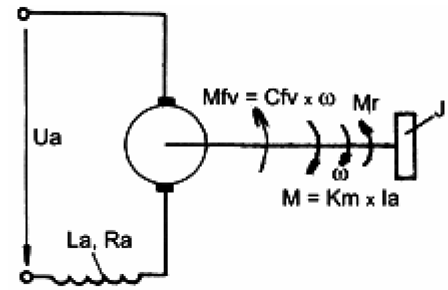

Fig. 5 Feed mechanism simplified schema

If M - (M_r + M_{fv}) ≠ 0, then the driving speed is increasing or decreasing, according to motor torque is greater or lower compared to feed mechanism couple. The difference between two couples is in fact the dynamic couple:

$$M - M_r - M_{fv} = J \cdot \frac{d\omega}{dt}, \qquad (30)$$

or

$$M - M_r - C_{fv} \cdot \omega = J \cdot \frac{d\omega}{dt} \qquad (31)$$

Differential equations that describe the dynamic of this servo-mechanism were obtained in base of Khirkoff second law, taking in consideration that motor excitation is constant and permanent. After writing the equilibrium equation of moments at driving shaft the following relations resulted:

$$L_a \cdot \frac{di_a}{dt} + R_a \cdot i_a = u_a - K_e \cdot \omega \qquad (32)$$

$$J \cdot \frac{d\omega}{dt} + C_{fv}\omega = K_m i_a - M_r \qquad (33)$$

$$\frac{d\varphi}{dt} = \omega \qquad (34)$$

In (32- 34) the significance of terms is:

u_a - motor supply voltage;

i_a - amperage through supply circuit;

ω - motor shaft angular speed (at ball screw shaft);

R_a - winding electrical resistance measured at brushes;

L_a - rotor inductivity;

u_e - electromotive voltage;

K_e - motor voltage constant;

K_m - motor couple constant;
J - equivalent inertia moment (reduced) at motor shaft;
C_{fv} - viscous frictional couple constant
M_r - resistant moment.

c. Input-output model of electromechanical feed mechanism

In order to study the dynamics of an electromechanical feed mechanism was utilised an input-output model. This is a representation compatible with numerical calculus methods. To represent this automated feed mechanism in steady space a vector x(t) was used to define the system state in every moment. In these conditions, input-output model for this mechanism can express with the following relations:

$$\frac{d}{dt}x = A\cdot x(t) + B\cdot u(t) + B_g; \qquad (35)$$

$$z(t) = C^T \cdot x(t) \qquad (36)$$

where (35) is state equation and (36) is output equation and
x(t) - steady vector
$u_a(t)$ - input value (voltage supply)
$\omega(t)$ - output value (driving shaft angular speed) – electrode position.

For constructing the input-output model, state characteristic values were selected and differential equations were laid under vector - matrix form (37):
$x_0 = i_a$; $x_1 = \omega$; $x_2 = \varphi$

$$\frac{d}{dt}\begin{vmatrix} i_a \\ \omega \\ \varphi \end{vmatrix} = \begin{vmatrix} \frac{-R_a}{L_a} & \frac{-K_e}{L_a} & 0 \\ \frac{K_m}{J} & \frac{C_{fv}}{J} & 0 \\ 0 & 1 & 0 \end{vmatrix} \cdot \begin{vmatrix} i_a \\ \omega \\ \varphi \end{vmatrix} + \begin{vmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{vmatrix} \cdot u_a + \begin{vmatrix} 0 \\ \frac{-M_r}{J} \\ 0 \end{vmatrix}, \qquad (37)$$

In equation (37), as variable were picked amperage from differential equation that dynamically describe the control circuit, angular speed and driving shaft rotation angle from equation which describe dynamic equilibrium at motor shaft level.
In (38), (39), (40) and (41) A represent the matrix coefficients for differential equations which are describing mechanism dynamic. B represents control matrix and C the output matrix.

$$A = \begin{vmatrix} \frac{-R_a}{L_a} & \frac{-K_e}{L_a} & 0 \\ \frac{K_m}{J} & \frac{C_{fv}}{J} & 0 \\ 0 & 1 & 0 \end{vmatrix}, \qquad (38)$$

$$B = \begin{vmatrix} \frac{1}{L_a} \\ 0 \\ 0 \end{vmatrix}, (39) \quad B_g = \begin{vmatrix} 0 \\ \frac{-M_r}{J} \\ 0 \end{vmatrix}, (40), \quad = \frac{d}{dt}\begin{vmatrix} i_a \\ \omega \\ \varphi \end{vmatrix} \qquad (41)$$

Modelling in this way the feed mechanism allows analysing its dynamic behaviour.
Trials were performed with to goal to investigate this mechanism performance under ideal and real conditions. In this second case, experiments were conducted in several situations (taking in account frictions, bearing clearances, several kinematics etc.). With this occasion were emitted several criteria imposed to electro discharge feed mechanism machines, especially for die sinking ones. The results are extremely important for designing and subsequent for correct operation of such an EDM machine.

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