A GENERAL MODEL OF MATERIAL REMOVAL AND SURFACE ROUGHNESS FOR SHORT PULSES AND LONG PULSES OF ELECTRIC DISCHARGE MACHINING

Seyed Hamid EBRAHIMI[1], M. GHOREISHI[2]

ABSTRACT

The electric discharge machining (EDM) process provide one of the best alternatives, or sometimes the only alternative, to machining a growing number of high-strength and corrosion and wear-resistant material used in a broad spectrum of industries. It has reported that aerospace or aerospace related industries own 40 percent of EDMs. In electro discharge machining (EDM), melting is main process for metal removal. It is known that EDM is basically a thermal process. Heat generated in discharge channel, melts end even evaporates the materials. However, for short pulses (discharge duration <5µs) melting does not account for the result as shown by the experiments reported. For short pulses, metal does not get enough time to get adequately heated and almost no melting take place. The electro static force acting on the surface is a very important factor in the removal of metal for short pulses. In the proposed, effect of electrostatic force and melting has been combined and the 'crater depth' has been calculated. It can be used for both short pulses and long pulses hold in al working conditions.

KEYWORDS: Electric discharge machining, electro static force, heat transfer

1. INTRODUCTION

In the EDM process, a discharge takes place between the electrodes. The dielectric between the electrodes break down and a spark channel is formed.

During the discharge, the whole spark region is in the plasma state. The plasma consists of atoms dissociated into positive ion and electrons and is highly conducting.

The ion density is nearly equal to the electron density and the plasma, to large extend, is electrically neutral. It is known that EDM is basically a thermal process.
Heat, generated in the discharge channel, melt and even evaporates the electrode materials. However, for short pulse (discharge duration <5µs), melting does not account for the result as shown by the experiments reported.

For short pulses, metal does not get enough time to get adequately heated and almost no melting take place. The electro static force acting on the surface is a very important factor in the removal of metal for short pulses.

2. Determination of the electro static force and effect of electro static force on the material removal rate

The voltage difference between the electrodes (of the order 10^2 V) cannot be accommodated in this neutral plasma region and a very thin sheath is formed near the negative electrode. The sheath has a charge imbalance and can sustain a potential gradient. This allows almost all the potential drop between the electrodes to take place in this sheath [Fig. 1(a)]. The whole plasma region can be considered an equipotential region at potential ϕ_p. The plasma potential is actually controlled by the positive electrode potential and differs only slightly from it. So with respect to the plasma potential, the cathode (the negative electrode) is at the negative potential. In the present study, the plasma potential is taken as the reference potential (zero potential). Because of this large potential drop in the thin sheath near the electrode, there is very strong electric field at the cathode. The field induces a negative charge on the cathode surface. The negative charge on the surface is pulled outwards by the field, which leads to a stress distribution on the surface and in turn, inside the metal. The discharge also causes heating of the metal. Due to heating, the yield strength of the metal decrease and the metal yields easily to the stress acting on it. During the process, for short pulse, the metal does not get enough time to get heated. Therefore, the effect of temperature is not visible for short pulses. However, for medium pulses (discharge duration between 5 and 100μs), temperature plays an important role in decreasing the yield strength of the metal and hence, increasing the depth of the crater formed. For long pulse, the spark radius becomes very large and as a result, the stress acting on the surface becomes very small. Therefore, the temperature becomes the key factor and the electro static force does not play a significant role.

1-2. Assumptions

1. The electrode is a semi-infinite zone.
2. The spark is a uniform, circular heat source on the electrode surface and the diameter of this source is constant.
3. Except for the yield strength, the properties of material do not change with temperature.
4. The rate of heat input remains constant throughout the discharge time.
5. The plasma region acquires the potential of the anode and the whole potential drop is in the sheath near the cathode and the plasma region near the sheath. The plasma potential is actually governed by the positive electrode potential and differs only slightly from it.
6. The metal yield at the end of the discharge.
7. In the spark channel, there is no variation in the potential, ion and electron densities and the ion velocity in the radial direction.

3. Estimation of the electric field

3.1. Analysis in the plasma region

The plasma region it self cannot accommodate the potential drop between the electrodes and a thin layer of charge is formed between the cathode and plasma. This layer is commonly termed as plasma sheath. Most of the potential drop takes place in the sheath. However, there is small potential in the plasma region as well and small pre-sheath electric field extends into the plasma. This pre-sheath electric field accelerates the ion such that when they inter the sheath, they acquire a drift velocity equal to the ion acoustic speed. Plasma fluid equation is applied in the plasma region. The plasma fluid equation (this is very similar to the Navier-Stokes equation of fluids) in the one dimension, with the electrostatic force is [1]:

$$m_i n\frac{\partial V}{\partial t}+m_i V\frac{\partial n}{\partial t}=-m_i V\frac{d}{dx'}(nV)-m_i nV\frac{dV}{dx'}-\frac{d}{dx'}(nkT_i)+enE \quad (1)$$

Where x' is in the direction of the voltage gradient [Fig. 2(b)] with the origin at reference point, taken at infinity (far away from the sheath) and n is the common density of ion and electrons, generally termed as plasma density:

$n = n_i = n_e$, n_e = electron density, n_i = ion density, V = velocity of ions in x' direction, m_i = the mass of an ion, T_i = the ion temperature, K = Boltzmann's constant, e = the electric charge ($1.6 * 10^{-19}$ Coulomb) = the total charge on an ion, E = the electric field at a point

The equation of mass conservation is as follows:

$$\frac{\partial n}{\partial t}+\frac{d}{dx'}(nV)=S(x') \quad (2)$$

Where S is the volume source rate of ion pairs (ion pairs/m^3/s). S can be, for instance, the local ionization of the neutral atoms and can also vary with x'. Substituting Eq.(2) in the plasma fluid equation and considering isothermal and steady-state flow, we get

$$nm_i V\frac{\partial V}{\partial x'}=-KT_i\frac{dn}{dx'}+enE-m_i VS \quad (3)$$

We have assumed the flow to be isothermal i.e. the ion temperature is constant in the whole discharge region. The electrons being in a retarding field, can be expressed by Boltzmann distribution:

$$n_e(x')=n_{\circ}\exp[e\phi/kT_e]=n(x') \quad (4)$$

Where $n_{\circ}$ is the reference electron density (or ion density) at infinity and T_e is the electron temperature. From Eq.(4)

$$enE=-en\frac{d\phi}{dx'}=-KT_e\frac{dn}{dx'} \quad (5)$$

Where ϕ is the potential at any point. So the final momentum equation can be written as

$$V\frac{dV}{dx'} = -\frac{C_s^2}{n}\frac{dn}{dx'} - \frac{SV}{n} \qquad (6)$$

Where $C_S = \left[(KT_i + KT_e)/m_i\right]^{1/2}$ is termed as the ion acoustic speed at the sheath/plasma interface. Taking $M(x') = V(x')/C_S$, with the ion acoustic speed as the normalizing speed and dividing Eq. (6) by C_S^2, we get

$$M\frac{dM}{dx'} = -\frac{1}{n}\frac{dn}{dx'} - \frac{SM}{nC_s} \qquad (7)$$

From Eq. (2), considering steady-state flow and taking $M = \frac{V}{C_S}$ we get

$$\frac{d}{dx'}(nM) = \frac{S}{C_s} \qquad (8)$$

Or

$$\frac{1}{n}\frac{dn}{dx'} = \frac{S}{MnC_s} - \frac{1}{M}\frac{dM}{dx'}$$

Substituting this in Eq. (7), we get

$$M\frac{dM}{dx'} = -\frac{S}{MnC_s} + \frac{1}{M}\frac{dM}{dx'} - \frac{SM}{nC_s} \qquad (9)$$

Or

$$\frac{dM}{dx'} = \frac{S}{nC_s} \times \frac{1+M^2}{1-M^2}$$

Since $\left(S/nC_S\right)$ is always positive and the flow starts with almost zero velocity (M<1) at the reference point, $\frac{dM}{dx'}$ is always positive. This means that the ions accelerate as the drift towards the cathode surface. As the Mach number approaches unity, the acceleration approaches infinity and the velocity is abruptly increased to a very high value. Physically this corresponds to the end of the plasma region and the beginning of sheath. The above equation can be solved easily for the plasma density variation with the Mach number [2].

$$\frac{n(M)}{n_\circ} = \frac{1}{1+M^2} \qquad (10)$$

From Eq. (5)

$$en\frac{d\phi}{dx} = KT_e\frac{dn}{dx}$$

Or,

$$\left(\frac{e}{KT_e}\right)d\phi = \frac{dn}{n}$$

Or,

$$\left(\frac{e}{KT_e}\right)\phi = \ln(nC)$$

Where C is the constant of integration. At the reference point, $n = n_\circ$ and $\phi = 0$ (the reference potential is taken as the plasma potential at infinity end it is taken as zero.

So, our calculation of the potential distribution inside the sheath is actually the potential relative to the plasma potential.) give

$C = \left(\frac{1}{n_\circ}\right)$. So

$$\phi(M) = \left(\frac{KT_e}{e}\right)\ln\left(\frac{n(M)}{n_\circ}\right) = -\left(\frac{KT_e}{e}\right)\ln(1+M^2) \qquad (12)$$

At the sheath/plasma interface, $M = 1\ (V = C_S)$. So, at the sheath/plasma interface, the ion density, $n = n_{se} = \frac{1}{2n_\circ}$ and the voltage, $\phi_\circ = -\left(\frac{KT_e}{e}\right)\ln 2$. This plasma sheath analysis gives us the potential at the sheath/plasma. The remaining potential drop is in the sheath.

3.2. Analysis in the sheath region

Since the sheath thickness is very small, we can neglect the ion formation in the sheath. Also, we assume that the ion temperature, $T_i = 0$, so that all the ions have velocity Cs when they enter the sheath. This allows use of the momentum equation for individual particles inside the sheath [Fig. 1(b)]. The momentum equation for individual particles is [3]:

$$m_i V \frac{dV}{dx} = eE = -e\frac{d\phi}{dx} \quad (13)$$

Or,

$$m_i V^2 + 2e\phi = cons\tan t = 0$$

where x is the direction of the voltage gradient with the origin at the sheath/plasma interface. The total energy is the same as that at the reference point and is equal to zero (at the reference point, the potential and the velocity of an ion are zero). At the sheath/plasma interface, the ion velocity is Cs. So, from Eq. (8), we get,

$$\phi_\circ = \frac{-\frac{1}{2} m_i C_s^2}{e} \quad (14)$$

Now, the ion current density going into the wall is constant throughout the sheath

$$j^+ = en_i(x)\, V(x) = en_{se} C_s \quad (15)$$

So, we get,

$$n_i(x) = \frac{n_{se} C_s}{V(x)} \quad (16)$$

From Eq. (8), we get $V = \sqrt{-2e\phi/m_i}$ and $C_S = \sqrt{-2e\phi_\circ/m_i}$. Using these values of V and Cs in the above equation, we get,

$$n_i(x) = n_{se}\left[\phi_\circ/\phi(x)\right]^{1/2} \quad (17)$$

Where n_{se} = ion density = electron density at the sheath/plasma interface (inside the plasma, both ion and electron density are equal till the beginning of the sheath, to a large extent). The electron distribution in the sheath can be assumed to be the same as Boltzmann distribution:

$$n_e(x) = n_{se}\, \exp[e(\phi - \phi_\circ)/KT_e] \quad (19)$$

Using these ion and electron densities in Poisson's equation

$$\nabla^2\phi = -\frac{e}{\varepsilon_\circ}(n_i - n_e) \quad (20)$$

Taking $y = \frac{dy}{d\phi}$, the above equation can be expressed as:

$$y\frac{dy}{d\phi} = \frac{e(n_\circ - n_i)}{\varepsilon_\circ} \quad (21)$$

Solving the equation, we get

$$y^2 = E^2(x) = \frac{2}{\varepsilon_\circ} n_{se} e\,\{2(\phi\phi_\circ)^{1/2} + (KT_e/e)\exp[e(\phi - \phi_\circ)/KT_e]\} + C \quad (22)$$

where *E* is the electric field at any point inside the sheath $\left(E = -\frac{d\phi}{dx}\right)$. To determine *C*, we use the boundary condition that at the sheath/plasma interface, the electric field, *E*(0) =0 and the voltage $\phi = \phi_\circ$.

The assumption that the field is zero at the sheath/plasma interface is quite valid because the voltage drops only by $\phi_\circ$ over a large distance (relatively infinite) and the electric field at the interface is much smaller than the electric field inside the sheath. So the final expression for the electric field is given by:

$$E^2(x) = \frac{2}{\varepsilon_\circ} n_{se} e\,\{-2\phi_\circ[(\phi/\phi_\circ)^{1/2} - 1] + (KT_e/e)[\exp(e(\phi - \phi_\circ)/KT_e) - 1]\} \quad (23)$$

The electric field at the electrode can be found out by substituting the cathode potential, ϕ_W, in the above equation (this is the potential relative to the reference potential or the plasma potential). The plasma potential has been assumed to be the same as the positive electrode potential. So ϕ_W is equal to the negative of the voltage applied between the electrodes.

Now, since $\left|e(\phi_W - \phi_\circ / KT_e)\right|$ is very large, $(KT_e/e)[\exp(e(\phi - \phi_\circ)/KT_e)$ can be neglected. Also, $\phi_\circ = \frac{-1}{2}\frac{m_i C_S^2}{e} = \frac{-1}{2}\frac{KT_\circ}{e}$ for *T*i =0. So Eq. (23)gets reduced to

$$E^2(x) = \frac{e}{\varepsilon_\circ} n_{se} e[2(\phi_W \phi_\circ)^{1/2} + 4\phi_\circ] \quad (24)$$

In the above equation the second term on the r.h.s. is much smaller in comparison to the first term as the electrode potential, ϕ_W is much larger than $\phi_\circ$. Hence, we can ignore the second term in the equation. The equation now becomes

$$E^2(x) = \frac{4}{\varepsilon_\circ} n_{se} e\sqrt{\phi_W \phi_\circ} \quad (25)$$

4. Determination of stress

Choosing a suitable surface and applying Gauss' law, we get (Fig. 2)

$$\oint_s E.ds = q/\varepsilon_\circ \quad (26)$$

where q is the total charge enclosed. So, from Fig. 2, it can be seen that the surface charge density is $\varepsilon_\circ E$ which results in a surface stress, $\sigma = \varepsilon_\circ E^2$. Hence, from Eq. (25), we get

$$\sigma = 4n_{se} e(\phi_\circ \phi_W)^{1/2} \quad (27)$$

Now applying Eq. (15) at the sheath/plasma interface, we get

$$j^+ = n_{se} e C_s \quad (28)$$

For large negative ϕ_W the current due to the electrons is very small and the current due to the ions can be assumed to be equal to the net current. Thus,

$$j^+ = j^{net} = (I/\pi R^2) \quad (29)$$

where R is the spark radius and I is the total current in the discharge. For I <13.5 A and the discharge duration $t_d < 10\ \mu s$, the spark radius R can be expressed as [4]:

$$R = R_\circ + kt \quad (30)$$

where $R_\circ = 5\times10^{-6}\ m$, $K = 3\times10^{-6}\ m/\mu s$ and t = time after the formation of the spark. From Eqs. (27)–(29), the expression of the stress becomes

$$\sigma = 4(I/\pi R^2)\frac{(\phi_\circ \phi_W)^{1/2}}{C_s} \quad (31)$$

Substituting $C_S = \left(-2e\phi_\circ/m_i\right)^{1/2}$ obtained from Eq. (12), we get

$$\sigma = 2\sqrt{2}\,(I/\pi R^2)\sqrt{m_i}\,(-\phi_W/e)^{1/2} \quad (32)$$

This is the stress acting on the electrode surface in the discharge area and has been implicitly assumed to be constant in the whole discharge region since the plasma and the sheath equations were solved for the one-dimensional case. The stress at any point A, with co-ordinates (ρ, z), due to a point force, *P*, on the surface is given by [Fig. 3(a)]

$$\sigma_z(r,z) = \frac{3pz^3}{2\pi(z^2+\rho^2)^{5/2}} \quad (33)$$

So, the stress at any point (*r*, *z*), inside the electrode, due to this circular distribution of stress, [Fig. 3(b)] is

$$\sigma_z(r,z) = \int_0^R \int_0^{2\pi} \frac{3\sigma z^3 r' d\theta\, dr'}{2\pi(z^2 + r^2 + r'^2 - 2rr'\cos\theta)^{5/2}} \quad (34)$$

The maximum stress is along *r* = 0, and can be found out using the above equation

$$\sigma_z(0,z) = \sigma[1 - \frac{z^3}{\sqrt{(z^2+R^2)^3}}] = \sigma[1 - (1 + (R/z)^2)^{-3/2}]$$

$$= \sigma[1 - (1 - \frac{3}{2}(R/z)^2)] \quad for (R/z)^2 << 1 \quad (35)$$

$$= \frac{3\sigma(\pi R^2)}{2\pi z^2}$$

stituting σ obtained from Eq. (16), we get

$$\sigma_z(0,z) = \frac{3I\sqrt{2m_i(-\phi_W/e)}}{\pi z^2} \quad (36)$$

At *r* = 0, $\tau_{rz} = 0$ by symmetry. So the Von-Mises stress is $\sigma' = \sqrt{\sigma_z^2 + 3\tau_{rz}^2} = \sigma_z(0,z)$. From Eq. (36), we can see that for $(R/Z)^2 << 1$, the stress at any point is independent of spark radius and the surface stress acts as a point force. Therefore, as the spark radius increases, the stress in the inner region remains unchanged as long as the spark radius is small compared to the crater

depth, z. This happens in the case of short pulses where the spark radius always remains small compared to the crater depth. However, for long pulses, the spark radius becomes large after sometime and the stress in the region where yield takes place, is not independent of the spark radius.
As the spark radius R grows, the stress inside the metal remains constant in the initial part of the spark and starts decreasing when R becomes comparable to z. So the yielding is likely to take place sometime in the middle of the discharge.

5. Determination of the yield strength

For yielding, the Von-Mises stress should be greater than the yield stress. Now, following the assumptions, the temperature inside the electrode at r = 0, at the end of the discharge is [2]:

$$T(0,z,t_d) = T_\circ + \frac{2H\sqrt{\alpha t_d}}{\pi k a^2 t_d}[ierfc(\frac{z}{2\sqrt{\alpha t_d}}) - ierfc(\frac{\sqrt{z^2+a^2}}{2\sqrt{\alpha t_d}})] \quad (37)$$

Where T_0= the initial temperature of the electrode, H= the total heat going into the electrode, a= the radius of the circular heat source, K= thermal conductivity of the metal, α = thermal diffusivity of metal, t_d the discharge duration
The yield strength varies with the temperature and can be expressed as a function of temperature. A fourth order curve can be fitted to match the yield strength-temperature curve for a metal. Taking yield stress to be zero at the melting point, we get a curve [5]

$$S_y(T) = A_\circ + A_1 T + A_2 T^2 + A_3 T_3^{\,3} + A_4 T_4^4 \quad (38)$$

Where $S_y(T)$ is the yield strength at temperature T and A_1, A_2, A_3, A_4 are constants. Since temperature inside the electrode is a function of r, z, and t_d, the yield strength is also a function of r, z and t_d.

6. Determination of the crater depth

Yielding occurs when the Von-Mises stress is more than the yield strength. So equating $S_y(T)$ or $S_y(r,z,t_d)$ to the Von-Mises stress at r = 0 gives the crater depth [6] (Fig. 4).

$$S_y(0,z,t_d) = \frac{3I\sqrt{2m_i(-\phi_W/e)}}{\pi S_y(0,z,t_d)} \quad (39)$$

Or,

$$z = \left(\frac{3I\sqrt{2m_i(-\phi_W/e)}}{\pi S_y(0,z,t_d)}\right)^{1/2} \quad (40)$$

7. Distributed point sources for Determination of heat transfer

Main assumption in this model is that the electrode surface intersecting the spark channel is covered by point sources. Total energy released by these sources is equal to total spark energy. Both of the spark channel and radius can be taken as time dependent in most general case.
Physically, the model can be visualized on atomic scale as an electron (or ion) bombardment on the electrode surface. Negatively charge electrons are strongly attracted towards the cathode. Under the high electric field the electrons and ions move at high speeds and strike the molecules of materials. They release their kinetic energy on the surface upon impact. The energy converted in to heat energy and dissipated in the material. This process continues as long as the spark exits. The amount of energy release by each of these electrons and ion depends on the electric field conditions. In some material, more energy is carried by the electrons (or ions) and this property determines it is applicability as a tool electrode material in EDM.
In continuum, each of these electrons (or ions) can be considered as an instantaneous point heat source. The total power is then equal to the total spark power.
For a practical solution, it will be assumed that the location of point source are fixed with time. In other words, each point source act for a certain time interval. In this case, temperature distribution is [7]

$$T(r,t) = \frac{(\rho c)^{1/2}}{8(\pi k)^{3/2}}\int_0^{t_s} Q(\tau)(t-\tau)^{-3/2}\exp\left[-\frac{r^2}{4\alpha(t-\tau)}\right]d\tau \quad (41)$$

8. Superposition of point source:

The governing differential equation for the above continuous point heat source temperature distribution is:

$$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r}\frac{\partial T}{\partial r} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \quad (42)$$

In spherical coordinates. The boundary conditions are,

$$1.\ \frac{\partial T}{\partial z}=0 \quad on\ \ z=0 \quad except\ r=0$$

$$2.\ Q(\tau)=Q(\tau)\delta(r)$$

If the heat source function is taken to be constant with time

$$Q(\tau)=\begin{Bmatrix} 0 \quad for\ any\ \ t,\ except\ at\ \ r=0 \\ Q\ \ at\ \ r=0 \quad for \quad 0\le t\le t_S \end{Bmatrix} \tag{43}$$

$$P(r,t)=\begin{Bmatrix} 0\ \ for\ \ any\ \ t,\ \ at \quad r>r_S \\ 0\ \ at \quad r<r_S \quad for\ \ 0<t<t_S,\ \ at \quad z=0 \end{Bmatrix} \tag{44}$$

In this case, the governing equation for any source (say i[th] source) will be [8]

And boundry conditions (at z=0)

$$\frac{\partial^2 T_i}{\partial r_i^{\,2}}+\frac{2}{r_i}\frac{\partial T}{\partial r_i}=\frac{1}{\alpha}\frac{\partial T_i}{\partial t_i} \quad \ldots\ldots\ldots\ldots \quad . \tag{45}$$

$$P_i(r_i,t)=\begin{Bmatrix} 0\ \ except\ at\ \ r=r_i\ \ and\ \ for\ \ 0<t<t_S \\ Q_i\ \ at\ \ r_i=0\ \ and\ \ for\ \ 0<t<t_S \end{Bmatrix}$$

(46)

Where r_i is the distance from i[th] point source to position P and t_i is the time such that t_i=0 when i[th] source started to act.

If cylindrical coordinates are chosen with point $(0,0,\psi)$ coinciding with the center of the circular source and radial distance are transformed to this system then, with the same time scale, the differential equations and boundary conditions can be summed up as follow:

$$\sum_{i=1}^{n}\frac{\partial^2 T_i}{\partial r^2}+\frac{2}{r}\frac{\partial T}{\partial r}=\frac{1}{\alpha}\sum_{i=1}^{n}\frac{\partial T_i}{\partial tq} \tag{47}$$

And boundary conditions become:

$$P(r,t)=Q\,u(t)\left[u(r)-u(r-r_S(t))\right] \quad at\ \ z=0$$
$$and \quad \frac{\partial T}{\partial z}=0 \quad for \quad r>r_S(t) \tag{48}$$

Then the differential equation is:

$$\frac{\partial}{\partial r^2}\sum_{i=1}^{n}T_i\ \frac{2}{r}\frac{\partial}{\partial r}\sum_{i=1}^{n}T_i=\frac{1}{\alpha}\frac{\partial}{\partial t}\sum_{i=1}^{n}T_i$$

$$or \tag{49}$$

$$\frac{\partial^2 \overline{T}}{\partial r^2}+\frac{2}{r}\frac{\partial \overline{T}}{\partial r}=\frac{1}{\alpha}\frac{\partial \overline{T}}{\partial t}$$

Where $\overline{T}=\sum_{i=1}^{n}T_i$, the temperature distribution due to the circular point source and T_i is due to the continues pulse point source. The summation sign is over the all source within the source area.

Writing explicitly, the temperature distribution becomes:

$$T(p,t)=\sum_{i=1}^{n}C\int_0^t\frac{Q_i(\tau)}{(t-\tau)^{3/2}}\exp\left[-\frac{R_i^2}{4(t-\tau)}\right]d\tau \tag{50}$$

Where R is the distance from each point source.

$$R^2=z^2+r_{si}^2+r^2+2rr_{si}\cos\psi \tag{51}$$

Here r and z are coordinates of the point P and r_s is the radial distance of the point source from z axis in cylindrical coordinates.

Assuming that r_{si} and Q_i are constant with respect to time, the integral in Eq. (50) can be taken, temperature distribution becomes:

$$T(P,t)=\frac{1}{2k\pi}\sum_{i=1}^{n}\frac{Q_i}{R_i}\,erfc\left[\frac{R_i}{2\sqrt{\alpha t}}\right] \tag{52}$$

In this formulation, it is possible to have different values for heat sources over the circular area. This may be necessary if the variation of the discharge current density (therefore heat source) from the discharge axis (z axis) is to be considered.

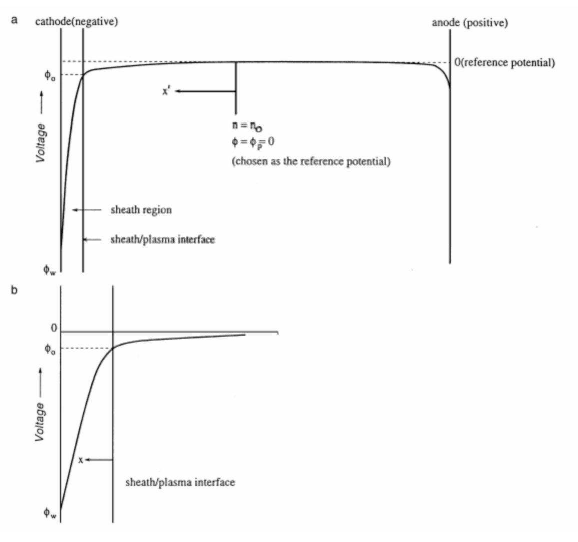

Fig. 1.(a) Voltage between the electrodes. (b) Voltage in the sheath region

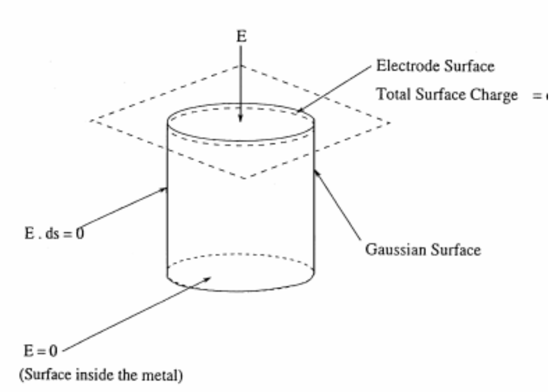

Fig. 2. Gaussian surface for surface charge distribution

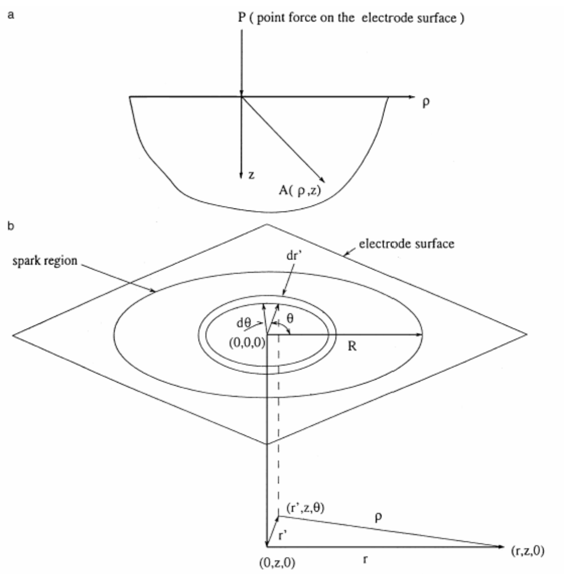

Fig. 3. (a) Stress due to point force P at any point A; (b) Stress at any point due to a circular stress distribution

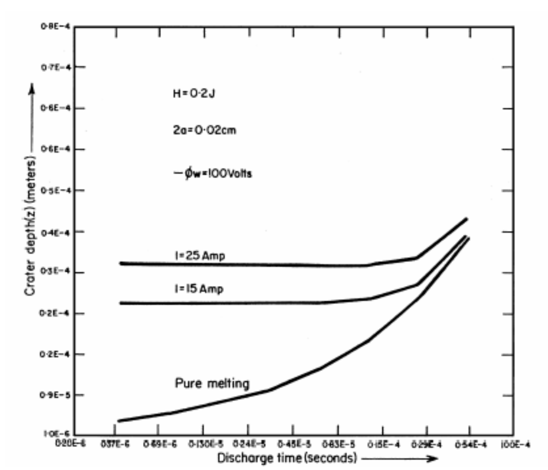

Fig. 4. crater depth variation with the discharge duration

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AUTHORS

[1] M.S graduate student Seyed Hamid EBRAHIMI, Dep. of Mech. Manufacturing & production. KNTU university, Tehran, Iran.
h_ebrahimi@sina.kntu.ac.ir

[2] Associate professor M. GHOREISHI, Dep. of Mech. Manufacturing & production. KNTU university, Tehran, Iran.