APPLICATIONS OF MONTE CARLO SIMULATION TO STRUCTURAL ENGINEERING PROBLEMS

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ABSTRACT: This paper investigates the application of Monte Carlo simulations in structural engineering to address various design optimization and uncertainty analysis. These applications include its uses in solving optimization problems and conducting uncertainty analysis. Three design problems are presented which are the design of a pressure vessel, the design of a rectangular welded beam, and the design of a compression spring. The performance of the Monte Carlo simulation along with other metaheuristic algorithms is compared numerically in the example problems. The comparison shows that the Monte Carlo simulation is a valid technique for structural design problems. The study concludes that despite its limitations, the simplicity and ease of implementation render Monte Carlo simulations an attractive option for structural design optimization scenarios where computational complexity may pose challenges.
KEYWORDS: Structural Engineering; Optimization; Monté Carlo; Simulation; Metaheuristic.

1. INTRODUCTION

The role of structural engineering is pivotal in shaping the built environment. It ensures the safety and reliability of civil infrastructure–buildings, bridges, towers, and dams–that support our daily lives. As technology continues to advance, structural engineers are faced with increasingly complex challenges to design and analyze structures subjected to an array of loads and environmental conditions. One of the powerful tools that has been gaining prominence in addressing these challenges is Monte Carlo simulation. This is a method for solving mathematical problems numerically using random sampling, often used as a computational technique in fields such as finance, physics, and, more recently, engineering.

A Monte Carlo simulation involves the generation of random samples to approximate numerical results and analyze the precise application of a system under consideration of uncertainty [1-8]. In the domain of structural engineering, variables and their inherently probabilistic nature (i.e., material properties, loading conditions, design parameters, etc.) render the probabilistic analysis using Monte Carlo simulation an invaluable method for evaluating many structural engineering problems. The paper provides a compilation of key structural engineering problems and their problem-specific manifestations of Monte Carlo simulation, which thereby gives direct insight into the application and implication of this method for the field.

The incorporation of Monte Carlo simulation in structural engineering practices provides a powerful platform for engineers to examine the probabilistic behavior of structures and make better decisions in their design and analysis processes. In addition to probabilistic analysis, Monte Carlo Simulation is a critical tool in structural engineering optimization problems. Optimization algorithms based on simulation allow engineers to seek the best possible design configuration for a structure to meet performance objectives with consideration of uncertainties and constraints. As such, it is able to explore a much larger set of design alternatives and therefore it is easier to identify robust solutions, i.e., those that are less sensitive to changes in operating conditions. This paper has addressed several examples of application of the Monte Carlo simulation technique in structural engineering showing their potential capability to address common structural design optimization problems and compared with commonly used metaheuristic algorithms.

At its core, an optimization problem consists of defining an fitness function to be optimized, along with a set of constraints that the solution must satisfy. The fitness function quantifies the goal, desired outcome, or the “quality” of the solution, while constraints represent the limitations or conditions that the solution must satisfy, adding a layer of complexity to the decision-making process. For example, a company might aim to maximize profits from a product subject to constraints on production, packaging, and shipping costs; an engineering team might seek to minimize energy consumption in a building given constraints on building materials, design, temperature, and environmental regulations; or a manufacturing company might look to optimize the design of a structure given a set of physical constraints.

An optimization problem can be mathematically defined as:

$$min/max\ f(X)$$

subjected to $g_k\ (X) \leq 0, for\ k = 1..n$

and $h_i\ (X) = 0, for\ i = 1..m$

where f is a function that is real-valued representing the optimization problem, g_k and h_i are both functions also real-valued representing the applied constraints, and X is a vector of decision variables that are to be manipulated by the optimization algorithm to achieve the best solution. The solution is then described as the vector X that satisfies all constraints (g_k and h_i) and optimizes the fitness function.

2. OPTIMIZATION ALGORITHMS

Optimization problems are addressed by a wide range of algorithms, each with its own set of advantages that is tailored to the specific characteristics of the problem space. Gradient-based methods like gradient descent use the gradient of the fitness function to perform iterative updates of decision variables, seeking to reach local minima or maxima. These algorithms work extremely well for smooth and continuous optimization landscapes. On the other hand, some optimization problems are best addressed by probabilistic algorithms, including those with a large number of local optima. Evolutionary algorithms draw inspiration from natural selection, with genetic algorithms and differential evolution as two key examples. They operate on populations of potential solutions, continually selecting "fitter" individuals, combining them by swapping and recombination, and mutating them to include some random solutions to prove valuable in particularly complex, multi-dimensional problems. Simulated annealing is a probabilistic transition between points in a space. Occasionally, it accepts solutions with poorer objective values as a means of making fewer, broad explorations of the space of solutions. This randomness helps avoid converging to local optima and make a clear path to the global optimal solution. Therefore, it seems that these sorts of probabilistic algorithms adapt well to a very active exploration of the space of solutions, mostly in non-convex and very irregular landscapes. Metaheuristic algorithms are a category of algorithms inspired by the collective movement in nature, in which a group of particles adjust their position based on personal and global best solutions including tabu search, genetic programming, ant colony optimization (ACO), and particle swarm optimization (PSO), provide adaptive and versatile strategies inspired by general problem-solving principles. ACO has been especially useful in scenarios where solutions are a selection from an almost infinite set of possibilities.

The selection of an optimization algorithm is influenced by the problem's characteristics - such as dimensionality, non-linearity, and the existence of constraints. Researchers typically try different algorithms or hybrid approaches, as they know that a given problem may need a different strategy for an optimum outcome. The diversity of algorithms only underscores the significance of choosing an approach suited to the peculiar challenges of the optimization problem in question.

3. MONTE CARLO SIMULATION PROCEDURE

Monte Carlo simulation is a computational technique named after the famous casino destination in Monaco, which is widely used in many fields. It leverages the power of statistical sampling. The technique involves repeated random sampling of input variables to solve a numerical problem. When we are confronted with optimization problems, Monte Carlo simulation is a very useful tool for dealing with uncertainties and variability in the fitness function and constraints. By injecting randomness in decision making, it allows us to explore a much larger potential space of solutions, which can be very helpful when traditional optimization methods are struggling due to the fitness function being complex, non-linear, or poorly understood. The procedure of the Monte Carlo simulation used in this study is as follows:

1. Start by defining the fitness function, decision variables, and constraints.
2. Generate random numbers for all decision variables within a suitable range.
3. Check if the generated random numbers satisfy the constraints. If yes, then go to step 4, otherwise go back to step 2.
4. Evaluate the fitness function at the randomly generated decision variables.
5. Steps 2 through 4 are repeated for the desired number of samples.
6. Perform statistical analysis on the result to find the optimum solution.

From the outlined procedure, it should be evident that Monte Carlo Simulation is an incredibly simple and yet, very powerful technique. The real difficulty in using it is the formulation of the fitness function and constraints, and the clarity and elegance of the procedure becomes clear when these elements have been properly designed.

The Monte Carlo method, however, is not void of limitation. Despite its versatility, there are some considerations that must be taken into account when using the Monte Carlo method alone as a method of optimization. Namely, it can suffer from high computational cost when used with very complex problems. This cost comes for the repeated evaluation of the fitness function this is especially true for high-dimensional problems. Another limitation of the Monte Carlo method is that it is inherently probabilistic. As such it does not guarantee a precise or global solution to the optimization problem. Nevertheless, it is still worth using for relatively simple optimization problem especially due to its ease of implementation.

4. STRUCTURAL DESIGN PROBLEMS

To evaluate the applicability of the Monte Carlo simulation to structural optimization problems, three typical benchmark problems were selected from the literature. The chosen problems include the design of a pressure vessel, a rectangular welded beam, and a compression/tension spring. These problems are commonly used as benchmarks in many studies and have been tackled using a variety of methods and techniques, including an augmented Lagrangian multiplier approach, co-evolutionary particle swarm optimization, enhanced ant colony optimization, evolution strategies, selection schemes, quantum-behaved PSO, cuckoo search, genetic adaptive search, a genetic algorithm-based co-evolution model, branch and bound techniques, and an integrated algorithm that combines particle swarm optimization with passive congregation [1]. The results of these approaches are presented in tables (1-3) and compared to the results of this study in the next section.

4.1 Optimization Problem 1: Design of Pressure Vessel

The pressure vessel to be designed is taken and its fitness function and constraints are from [2]. The vessel has a volume of 750 ft^3 and must withstand an operational pressure of 2000 psi. The selected shape of the vessel is cylindrical with two hemispherical heads placed at the ends. The aim is to reduce the overall expenses of the vessel, encompassing the expenses associated with fabrication, materials, and welding. The parameters involved in the problem revolve around the thickness of the main body of the pressure vessel x_1, the thickness of head two head x_2, the inner radius x_3, and the length of the vessel x_4. The fitness function is given by:

$$f(X) = 1.78\, x_2 x_3^2 + 0.6224\, x_1 x_3 x_4 + 3.1661\, x_1^2 x_4 + 19.84\, x_1^2 x_3 \quad (1)$$

$$g_1(X) = 0.01\, x_3 - x_2 \le 0 \quad (2)$$

$$g_2(X) = 0.02\, x_3 - x_1 \le 0 \quad (3)$$

$$g_3(X) = 129600 - \pi x_3^2 x_4 - 1.33\, \pi x_3^3 \le 0 \quad (4)$$

$$g_4(X) = -240 + x_4 \le 0 \quad (5)$$

And the search space for the variables is $0.06 \le x_1 \le 2$; $0 \le x_2 \le 6.2$; $10 \le x_3 \le 100$; $0 \le x_4 \le 200$.

4.2 Problem 2: Design of a Welded Rectangular Beam

The goal of this design challenge is to reduce the production cost of a welded cantilever beam shown in (**Error! Reference source not found.**) that is subjected to a load $P = 6000$ psi while adhering to constraints related to buckling load, end deflection, and shear stress. The rectangular beam has an overhang length $L = 14\, in$ and is made from steel with modulus elasticity $E = 30000$ psi, and shear modulus $G = 12 \times 10^6$ psi. The variables to be optimized are the thickness of the weld $h = x_1$, the length of the joint $l = x_2$, the thickness of beam $t = x_3$, and the width of beam $b = x_4$. This design problem, fitness function, and constraints are taken from [2, p. 431–433].

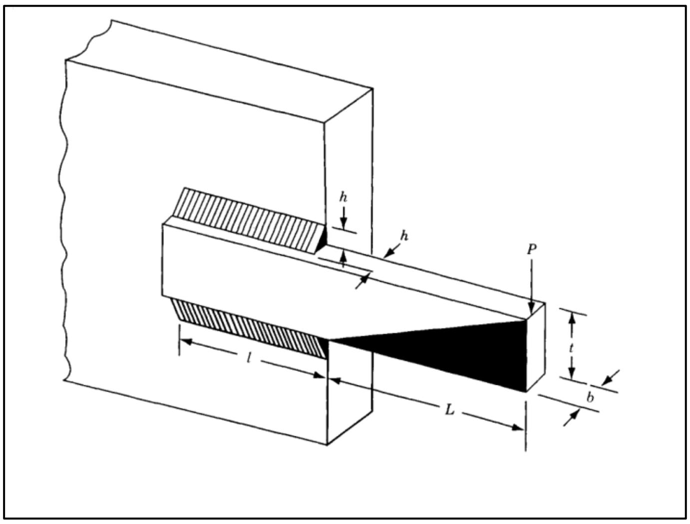

Figure 1. Welded beam geometry

The fitness function of the welded beam problem is given by:

$$f(X) = 1.1047\, x_1^2 x_2 + 0.048\, x_3 x_4 (14 + x_2) \quad (6)$$

Subjected to the constraints:

$$g_1(X) = -x_4 + x_1 \le 0 \quad (7)$$

$$g_2(X) = -\sigma_{max} + \sigma(X) \le 0 \quad (8)$$

$$g_3(X) = -\tau_{max} + \tau(X) \le 0 \quad (9)$$

$$g_4(X) = -x_1 + 0.125 \le 0 \quad (10)$$

$$g_5(X) = -\delta_{max} + \delta(X) \le 0 \quad (11)$$

$$g_6(X) = -P_c(X) + \mathrm{P} \leq 0 \quad (12)$$

And the search space for the decision variables is $0.1 \leq x_1, x_4 \leq 2$; $0.1 \leq x_2, x_3 \leq 10$. where τ is the shear stress within the weld, τ_{max} is the allowable shear stress = 13600 psi, P_c is the buckling load, P is the applied load to the beam = 6000 lb, σ is the normal stress in the beam, σ_{max} is the allowable normal stress in the beam = 30000 psi, δ is the end deflection of the beam, and δ_{max} is the maximum allowed end deflection = 0.25in. The shear stress τ, the normal stress σ, the end deflection δ, and the buckling load P_c are given by the relations:

$$\tau(X) = \sqrt{(\tau')^2 + (\tau'')^2 + \frac{1}{R}\tau'\tau''^{x_2}} \quad (13)$$

$$\sigma(X) = \frac{6PL}{x_4 x_3^2} \quad (14)$$

$$\delta(X) = \frac{4PL}{E x_3^3 x_4} \quad (15)$$

$$P_c(X) = \frac{4.013\sqrt{EG\left(\frac{x_3^2 x_4^6}{36}\right)}}{L^2}\left[1 - \frac{x_3}{2L}\sqrt{\frac{E}{4G}}\right] \quad (16)$$

Where:

$$\tau' = \frac{P}{\sqrt{2}x_1 x_2} \quad (17)$$

$$\tau'' = \frac{MR}{J} \quad (18)$$

And:

$$M = PL + 0.5Px_2 \quad (19)$$

$$= \sqrt{0.25x_2^2 + (0.5(x_1 + x_3))^2} \quad (20)$$

$$J = 2\left\{\sqrt{2}x_1 x_2\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)\right]\right\} \quad (21)$$

4.3 Problem 3: Design of Tension/Compression Spring

The spring design problem depicted in (Figure 2) is a constrained continuous problem. The aim is to optimize the volume (V) of a spring coil such that it is as small as possible under a constant tensile or compressive load. This problem involves three decision variables. The variables are the spring's active coils $N = x_1$, the diameter of windings $D = x_2$, and the mean diameter of spring wire $d = x_3$.

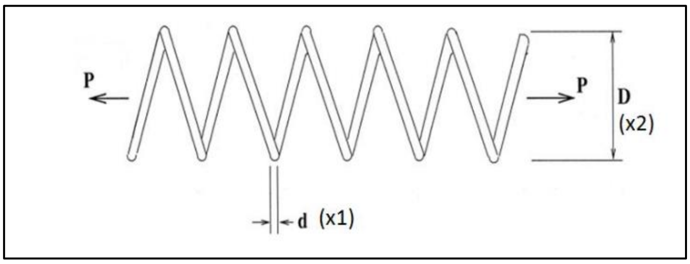

Figure 2. Spring geometry for the spring design problem

The fitness function to be optimized is taken from [4] and it is as follows:

$$f(X) = x_3^2 x_2 x_1 + 2x_3^2 x_2$$

Subjected to:

$$g_1(X) = 1 - \frac{1}{71785x_3^4}x_2^2 x_1 \leq 0$$

$$g_2(X) = -1 + \frac{1}{5108x_3^2} + \frac{4x_2^2 - x_3 x_2}{12566(x_2 x_3^3 - x_3^4)} \leq 0$$

$$g_3(X) = -\frac{140x_3}{x_1 x_2^2} + 1 \leq 0$$

$$g_4(X) = \frac{x_3 + x_2}{1.5} - 1 \leq 0$$

The search space for the decision variables is$2 \leq x_1 \leq 15; 0.25 \leq x_2 \leq 1.3; 0.05 \leq x_3 \leq 2;$

5. NUMERICAL RESULTS OF THE MONTE CARLO SIMULATION

The outcomes of the Monte Carlo simulation conducted for the three structural optimization issues are presented in (Figure 3-Figure 5). For each problem, a total of 1000 trials (samples) were conducted. The selection of the number of samples was such that it strikes a good balance between accuracy and speed, The optimal solution for the pressure vessel design problem is X=⟨1.0277,0.4965,49.6177,118.47216⟩ and the result of the fitness function is f(X)=6069.2396. checking the constrains $[g_1, g_2, g_3, g_4]$ they are equal to [-0.0701,-0.0231,-131970.3955,-121.5278] which indicates that the found solution is valid. Figure 3 displays the progression of Monte Carlo trials used to optimize the pressure vessel design problem.

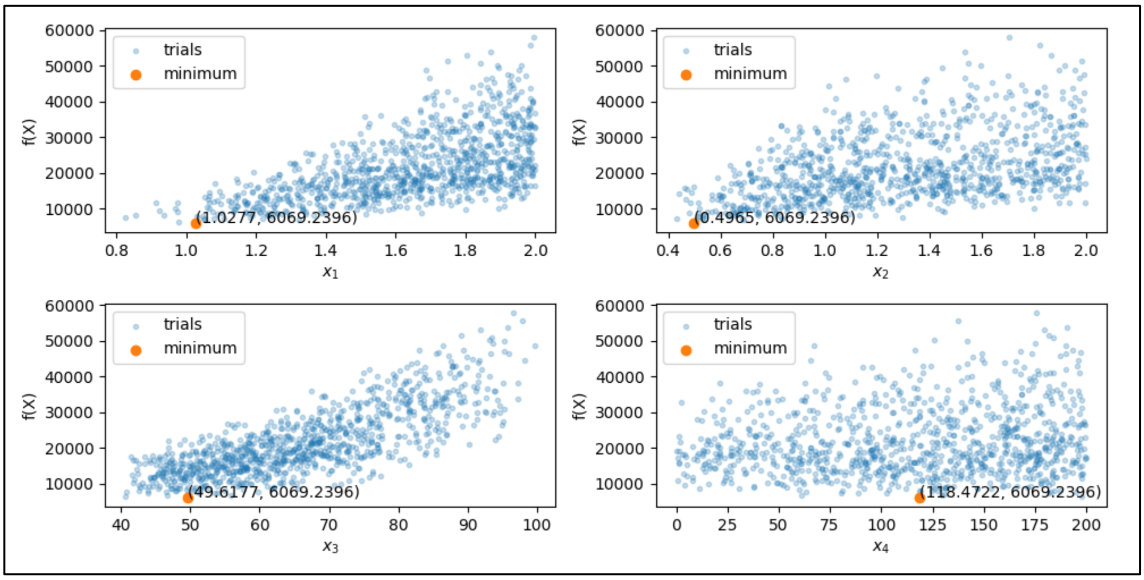

Figure 3. Monte Carlo trials applied to the pressure vessel design problem

The optimum solution of the found by the Monte Carlo simulation is shown in Table 1 along with the results obtained by other studies. Comparing the results obtained in this study to other methods shows a high level of similarity.

Table 1. Comparison of optimal solutions achieved using various methods for the pressure vessel design problem

Method	**x1**	**x2**	**x3**	**x4**	**f(X)**
Coelho [5]	0.8592	0.5221	42.1429	176.6191	6060.0471
Cagnina et al. [6]	0.8954	0.5012	42.1952	176.6367	6060.0517
Montes et al. [7]	0.9052	0.4531	42.1227	176.6606	6060.0454
Deb and Gene [1]	0.8854	0.5124	42.1671	176.6739	6060.0346
Kannan and Kramer [2]	0.8745	0.4464	42.1892	176.6872	6060.0904
He et al. [10]	1.1675	0.6378	47.7181	117.7856	8129.0853
Hu et al. [11]	1.1330	0.6615	58.3441	43.7079	7198.0119
Coello [12]	0.8503	0.4562	40.3273	200.0699	6289.0640
Kaveh and Talatahari [13]	1.1093	0.5939	49.6825	118.5722	6069.0435
Sandgren [14]	0.9929	0.5060	48.3539	112.7500	6410.0780
Coello and Montes [15]	0.9029	0.4491	42.1198	176.7678	6060.0138
Zhang and Wang [16]	0.9046	0.5291	42.1784	176.7723	6061.0559
Gandomi et al. [17]	1.1896	0.7097	58.3419	43.7270	7198.0002
Montes and Coello [18]	0.9054	0.4525	42.1976	176.6308	6060.0907
Kaveh and Talatahari [19]	0.8454	0.5099	42.1365	176.6314	6060.0624
Akay and Karaboga [20]	0.8620	0.4763	42.1783	176.6374	6060.0075
He and Wang [21]	0.8197	0.4956	42.1583	176.6984	6059.0641
This study	**1.0277**	**0.4965**	**49.6177**	**118.47216**	**6059.2396**

Similarly, the best solution for the welded beam problem found using the Monte Carlo simulation is $X = \langle 0.2500, 7.5590, 6.9732, 0.2578 \rangle$ with the value of the fitness function being $f(X) = 2.3866$. checking the constraints $[g_1,\ g_2,\ g_3,\ g_4,\ g_5,\ g_6,\ g_7] = [-164.7016,\ -3200.5075,\ -0.0079,\ -3.1286,\ -0.1250,\ -0.2499,\ -215.5455]$ indicates that the solution is satisfactory. A history of the Monte Carlo simulation for the welded beam problem is presented in **Error! Reference source not found.**.

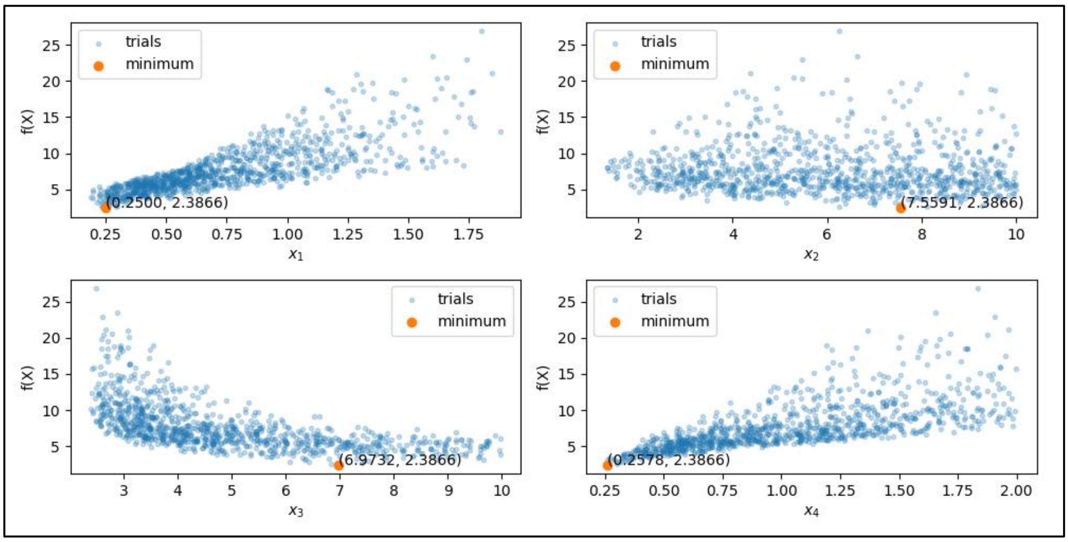

Figure 4. Monte Carlo trials used for the welded beam design problem

Table 2 compares the results of this study with those of other studies that employ different methods and approaches.

Table 2. Comparison of optimal solutions achieved using various methods for the welded beam design problem

Method	x1	x2	x3	x4	f(X)
Rao [3]	0.3152	6.2816	8.3410	0.2664	2.4796
Deb [22]	0.3078	6.2359	8.2602	0.2587	2.5301
Lee and Geem [9]	0.3093	6.3016	8.3491	0.2759	2.4281
Ragsdell and Phillips [23]	0.3191	6.2430	8.2799	0.2769	2.4846
Ray and Liew [24]	0.2695	6.2630	8.3837	0.2524	2.4665
Mehta and Dasgupta [25]	0.2982	6.2604	8.3495	0.2990	2.4415
Hwang and He [26]	0.2793	1.6060	12.9204	0.2521	2.2503
This study	**0.2500**	**7.559**	**6.9732**	**0.2578**	**2.3866**

Finally, the optimal solution for the spring design problem was identified through the Monte Carlo simulation as $X = \langle 0.0543, 0.4077, 9.6273 \rangle$ and the value of the fitness function was found to be $f(X) = 0.0139$. checking the constraints $[g_1,\ g_2,\ g_3,\ g_4] = [-0.0447,\ -0.0304,\ -3.7653,\ -0.6919]$ indicates a valid solution. Figure 5 illustrates the progression of Monte Carlo trials applied to the spring design problem..

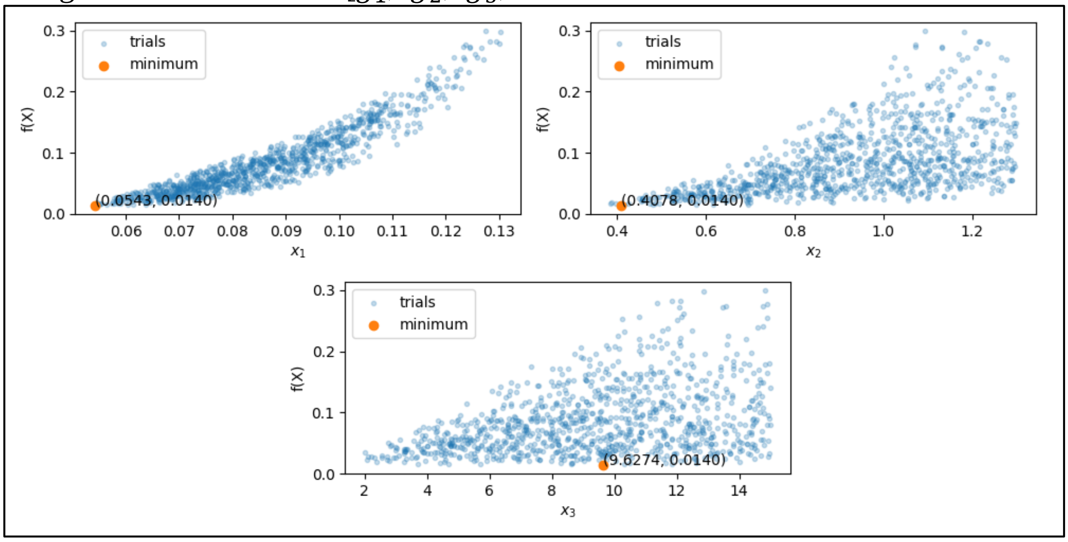

Figure 5. Monte Carlo trials applied to the spring design problem

Table 3 presents a comparison between the results of this study and the results of different methods.

Table 3. Comparison of optimal solutions achieved using various methods for the spring design problem

Method	x2	x1	x3	f(X)

He et al. [10]	0.3744	11.2936	0.0571	0.0155
Cagnina et al. [6]	0.3610	11.4409	0.0561	0.0209
Ray and Saini [14]	0.3260	13.9811	0.0583	0.0150
Keveh and Talatahari [19]	0.3622	11.0003	0.0607	0.0165
Zhang et al. [28]	0.3586	11.2973	0.0603	0.0208
Hu et al. [11]	0.3606	11.6135	0.0594	0.0186
Mahdavi et al. [29]	0.3547	12.0802	0.0608	0.0144
Coello [12]	0.3594	11.6376	0.0524	0.0198
Ray and Liew [24]	0.3750	10.6570	0.0575	0.0217
He et al. [10]	0.3589	11.2216	0.0546	0.0222
Omran and Salman [27]	0.3670	11.2469	0.0586	0.0140
Cagnina et al. [6]	0.3584	11.2062	0.0600	0.0157
Ray and Saini [14]	0.4002	9.1885	0.0596	0.0200
Keveh and Talatahari [19]	0.4128	8.6874	0.0586	0.0181
Zhang et al. [28]	0.3647	11.4057	0.0579	0.0149
Hu et al. [11]	0.3628	11.2954	0.0604	0.0151
Mahdavi et al. [29]	0.3663	10.8991	0.0612	0.0154
Coello [12]	0.3188	14.2548	0.0505	0.0220
Ray and Liew [24]	0.3529	11.5465	0.0561	0.0204
This study	**0.4077**	**9.6273**	**0.0543**	**0.0139**

6. CONCLUSION

The study examines how the Monte Carlo simulation can be used to tackle different optimization challenges in structural engineering design, including the design of a rectangular welded beam, design of compression spring, and design of pressure vessel. These optimization efforts seek to reduce design expenses while meeting a range of nonlinear constraints. These optimization tasks aim to minimize design costs while adhering to various nonlinear constraints. The biggest advantage of the Monte Carlo simulation is its ease of implementation, since the only requirement is the availability of the fitness function to be optimized and its corresponding constraints. To assess the effectiveness of the Monte Carlo simulation, numerical experiments are carried out and compared against alternative optimization approaches, particularly meta-heuristic algorithm-based methods. Tables (1-3) show that the solutions identified through the Monte Carlo simulation are similar to the best solutions identified by other methods for each problem, thus validating the approach to finding a solution. However, this method suffers from one major downside which is its sluggishness when selecting valid solution candidates. Since it relies heavily on randomness, the selected solution candidates do not always satisfy the constraints of the problem. This becomes more apparent as the number of constraints grows. Despite this, however, the simplicity and ease of implementation make Monte Carlo simulations an attractive option for certain structural design optimization scenarios where computational complexity may limit the use of more detailed models of the system.

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