

APPLICATION OF DIMENSIONAL ANALYSIS FOR MODELING MANUFACTURING PROCESSES: A REVIEW

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ABSTRACT: Dimensional analysis is a tool used to understand and analyze various problems in engineering. This analysis is useful for calculating dimensionless parameters and gives an answer as to which group of parameters affects the problem. Dimensional analysis can be represented using Buckingham's π-theorem. This paper provides an overview of the literature on the application of dimensional analysis and Buckingham's π theorem for various processing processes. The review period is observed in the period from 2007 until today. The main focus remains of the production process, performance and parameters of process, dimension and years of publication. Review work in this area aims to provide ready-made information in one place that can be very useful to future researchers in this direction of research.
KEYWORDS: Dimensional analysis, Buckingham's π theorem, production process.

1. INTRODUCTION

The progress of the industry and the development of science have conditioned the improvement and further development of manufacturing processes. Different manufacturing processes are widely developed and used with an ultimate aim provide high production rate and profit for manufacturers and at the same time to ensure high quality for customer. To this aim, it is of utmost importance to determine (near) optimal manufacturing conditions for a given combination of the workpiece material, machine tool and tool. Since most manufacturing processes is very complex, as a number of physical phenomena are involved, the usual industrial praxis is reflected in the fact that the choice of the main process parameters is usually based on the operator or engineering experience or is guided based on recommended conditions. The selected process parameters are rarely optimal, and do not ensure the maximal utilization of the given manufacturing technology. The development of physics-based models (analytical) can be helpful in understanding fundamental underlaying phenomena. However, since its definition is very time-consuming requiring interdisciplinary domain knowledge and often involve certain simplifications and approximations, the development of mathematical models for representing a given manufacturing process based on the experimental data is widely used in manufacturing practice. Based on the use of different an experimental matrix of different experimental design of experiments (DOE) experimental [39] matrix one obtain an efficient experimental hyperspace (in terms of possible production conditions) is obtained, which enables the development of various mathematical models. Moreover, such determined mathematical models can be used in line with CAM systems thus enabling intelligent process planning. Among different types of mathematical models used in manufacturing praxis classical regression models, power, response surface, artificial neural network, fuzzy logic models and models derived with the use of dimensional analysis are the most widely used [40].

Given that dimensional analysis is very useful for development of process models while considering a number of possible significant process variables, the purpose of this paper is to present the basic principles of dimensional analysis (DA) in one place, as an alternative method for deriving the mathematical models, and review the application of the DA for modeling of different manufacturing processes [43]. The search for papers was conducted using scientific resource bases, for last five years.

2. DIMENSIONAL ANALYSIS-BASIC PRINCIPLES

In the study, researchers create equations using mathematical analysis, resulting in analytical equations accurate for any system. This results in each group member having the same dimensional representation. This is the law of dimensional homogeneity used to form equations where the relationship of the variables is unknown. [1]. By using DA one can form a group and create a relationship between them. DA reduces the number of independent parameters and simply determines dimensionless criteria of similarity and relations. Independent parameters are represented as dimensionless groups that are included in the research problem of modeling process. In a modeling experiment, the main role of DA is to reduce the variables, to simplify the design. In the case where the mathematical model is not known, this method is particularly effective [2].

As a starting point for the application of DA, it is necessary to adopt the basic set of dimensions as a list of all factors involved. Factors in this case represent a set of dependent and independent variables and parameters that are relevant to the research problem, so they can be grouped into dimensionless entities. In fact, the factors represent all the quantities that will appear in the theoretical solution of the problem. Based on the dimension of each factor, the process recommends how the factors can be grouped into dimensionless entities [1, 3]. The main functions of DA are [1, 4]:

- determination the number and shape of dimensionless quantities
- reduction of independent variables in the experiment,
- simplification of the solution and generalization of its results,
- conversion the basic set of measurement units,
- converting quantities to other fundamental units,
- determining functionals when the solver does not know the detailed data of mathematical description.

Buckingham's π-theorem in DA determines the number of dimensionless entities required to find a relationship between them [5, 6]. The fundamental dimensions used in process models are mass (M), length (L), time (T), temperature (θ) and current (I) [13].

This theorem related to the dimensional method was first proposed by Lord Rayleigh (1877) [7], all variables that appear in a research problem can be grouped into a number of dimensionless π groups equal to number of variables (n) reduced by the number of dimensions (p). If dimensionless groups are represented as π_1, π_2, π_3, etc., then the equation expresses the relationship between variables whose solution is in the form, equation (1) or equivalently (2) [46].:

$$F(\pi_1, \pi_2, \pi_3, \ldots) = 0 \quad (1)$$

$$\pi_1 = f(\pi_2, \pi_3, \ldots) = 0 \quad (2)$$

3. APPLICATION OF DA FOR MODELING OF MANUFACTURING PROCESSES

The notion of DA is not a new concept. In one form or another, this has been widely practiced for a century. Most classical research used the idea of DA without referring to its name. In the early part of the twentieth century, analysis was finally dedicated to a systematic approach to the topic.

Several of them proposed a way with a different, not real dimension [1]. McWell introduced the basic units: mass, length and time [2, 42]. The first who tried to formulate a theory of DA was Jean B. Fourier [10]. Langhar [11] explained in detail the principles of DA for problem solving based on dimensionally homogeneous [44].

DA as a very powerful tool is used to model, analyse and understand various problems in engineering. The application of DA for modeling of different manufacturing processes is given in Table 1. The covered period spans from 2007 until today.

Table 1. Review of DA modelling manufacturing processes

Reference	Manufacturing process	Process performance	Dimensions used	Process parameter involved	Experimental matrix
[12]	AWJM	MRR Surface roughness	M, L, T, θ	Transverse speed Standoff distance Abrasive flow rate	Taguchi (25)
[13]	EDM	MRR	M, L, T, θ, I	Pulse on time, Pulse off time Average voltage Volume fraction	CCD (46)
[14]	EDM	Tool wear	M, L, T, θ, I	Average working voltage Discharge time Melting point Thermal expansion coefficient Electrical conduction Density Thermal conductivity Volumetric wear	Taguchi (27)
[15]	High speed cutting	Temperature	M, L, T, θ	Melting point Cutting speed Depth of cut Feed rate Specific cutting energy	Taguchi (27)

Reference	Manufacturing process	Process performance	Dimensions used	Process parameter involved	Experimental matrix
				Specific heat Density Thermal conductivity	
[16]	SLS	Surface roughness	M, L, T	Laser beam power Laser scanning speed Hatching Powder bed thickness	Taguchi (27)
[17] *	EDM	Tool wear	M, L, T, θ, I	Pulse off time Peak current Pulse on time Wire feed rate	Taguchi (18)
[18] *	EDM	MRR	M, L, T, θ, I	Pulse on time Pulse off time Peak current Wire feed Wire tension Servo voltage	Taguchi (18)
[19]	EDM	MRR Surface roughness	M, L, T, θ, I	Pulse off time Pulse on time Wire feed rate Peak Current	Taguchi (27)
[20] *	EDM	MRR Surface roughness	M, L, T, θ	Pulse off time Pulse on time Spark voltage Peak Current	Taguchi (18)
[21] *	EDM	MRR Surface roughness	M, L, T, θ, I	Powder concentration Pulse on time Spark voltage Peak Current Gap voltage	Taguchi (18)
[22]	SLM	Bulk density	M, L, T, θ	Volumetric energy density Average particle diameter Scanning speed Specific heat capacity Heat conductivity	Taguchi (9)
[23]	Milling	MRR	M, L, T, I	Cutting speed Feed Depth of cut	Taguchi (27)
[24] *	EDM	MRR	M, L, T, I	Pulse-on-time Pulse-off-time Wire feed rate Spark voltage Servo-speed	FFD (16)
[25] *	Drilling	Penetration rate	M, L, T	Weight on bit Bit diameter Bit rotational speed Weight on bit Fluid velocity Fluid density	Taguchi (18)
[26]	Drilling	Heat Generated	M, L, T, θ	Cutting Force Spindle Speed Feed Diameter Time from contact Temperature Thermal Conductivity	Taguchi (18)
[27]	AWJM	Surface roughness	M, L, T	Water pressure Traverse speed Mass flow rate	Taguchi (27)
[28]	EDM	MMR	M, L, T, I	Orbital radius Orbital speed Gap voltage Pulse ON time	Taguchi (25)
[29]	Boring	Tool wear	M, L, T	Cutting speed	FFD

Reference	Manufacturing process	Process performance	Dimensions used	Process parameter involved	Experimental matrix
*				Depth of cut Nose radius Cutting Time Spindle Load Coolant concentration Coolant pressure	(120)
[30]	Grinding	Force	M, L, T	Spindle speed Work speed Depth of cut Width of cut Tensile strength	Taguchi (8)
[31]	AWJM	Surface roughness	M, L, T, θ	Nozzle diameter Traverse speed Jet angle Standoff Pump pressure Mix mass ratio	Taguchi (18)
[32] *	Turning	Surface roughness Temperature Machine vibration	M, L, T	Cutting speed Feed Depth of cut	Taguchi (27)
[33] *	Burnishing	Surface roughness	M, L, T	Burnishing Speed Cutting Feed Cutting Fluid Circumferential area of material	FFD (80)
[34]	Turning	Surface roughness	M, L, T	Cutting speed Feed Depth of cut	Taguchi (27)
[35] *	EDM	MRR Surface roughness	M, L, T	Pulse on time Pulse off time Wire feed rate Servo voltage	Taguchi (27)
[36] *	USM	Tool wear	M, L, T	Slurry hardness factor Slurry grit size Elastic modulus of tool Power rating	Taguchi (18)
[37]	USM	MRR	M, L, T	Power rating, Tool hardness factor Slurry hardness factor Slurry grit size	Taguchi (27)
[38] *	Turning	MRR Power consumption	M, T, L	Cutting speed Feed Depth of cut	FFD (330)

AWJM (Abrasive Water Jet), SLS (Selective Laser Sintering), SLM (Selective Laser Melting), USM (Ultrasonic Machining,) MRR (material remove rate), M (mass), L (length), T (time), θ (temperature), I (current). FFD (Full Factorial Design), CCD (Central Composite Design).

4. RESULTS AND DISCUSSION

The number of experimental studies that involved the definition of different production process models based on the use of DA and Buckingham's π-theorem is a very small number based on the review, Fig. 1 shows their chronological application in the period from 2007 until today. Most papers were published in 2013, 2015 and 2020, there is probably a possibility that this number will increase in the coming years.

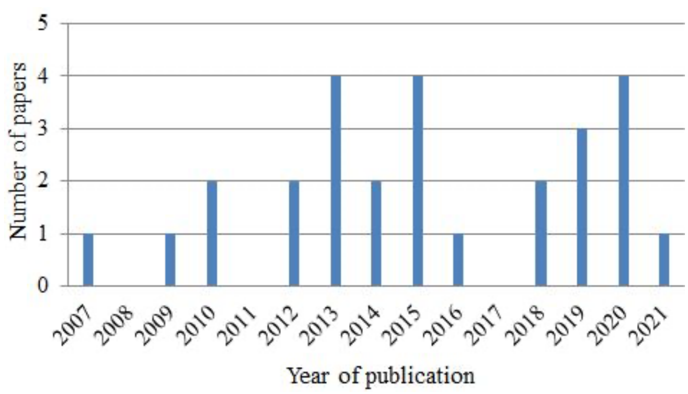

Figure 1. Yearly distribution of publications

Most of the work is related to modeling of EDM, while for other manufacturing processes there is only a couple or one, (Fig. 2). From this it can be observed that DA has not been applied for modeling deformation processing technologies and other non-conventional machining processes. Also, it is clear that there is a limited application for modeling of traditional machining processes. The majority of reviewed investigations were aimed at establishing relationships between process parameters and performances related to productivity, production quality and the actual process. The majority of reviewed investigations were aimed at establishing relationships between process parameters and performances related to productivity, production quality and the actual process.

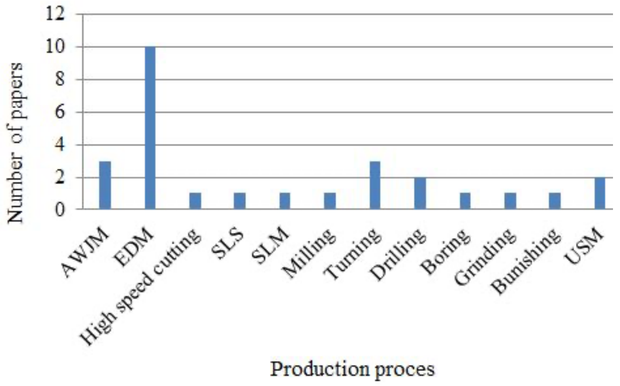

Figure 2. Number of papers related to modeling off different manufacturing processes

The performance of production processes that are mostly modeled by applying DA and Buckingham's π-theorem are MRR and surface roughness, (Fig. 3).

Process optimization was performed in papers marked with *.

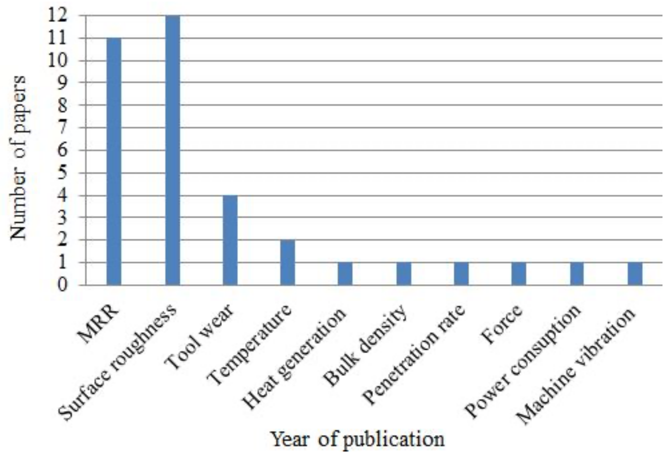

Figure 3. Performance modeled by using DA

5. CONCLUSION

This research presents a five-year review of Da for modeling various production processes. The aim was to identify in which manufacturing processes the DA was mostly used as well as to investigate which performances were taken into consideration which could later be used for future research and application of this analysis to other manufacturing processes. Also, the review was aimed to investigate the number of involved parameters in model development and which DOE matrix and corresponding number of trials were adopted for obtaining experimental data for model development. The review shows that there are limited modeling studies in machining processes based on the use of DA, and that there are no in deformation processing technologies and other non-conventional machining processes. The majority of reviewed studies developed mathematical models for relating the process parameters process characteristics related to productivity and product quality, wherein there are no studies related to modeling of other performance characteristics such as processing time, manufacturing costs, environmental aspects and other specific process characteristics.

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