

# THE PROCESSING OF PLANE COMPLEX CONTOURS USING WIRE ELECTRICAL DISCHARGE MACHINING

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**ABSTRACT:** The paper presents two original methods that can be used in the case of cutting complex contours using wire electrical discharge machining (WEDM): the osculating circle method and the three-point circle definition method. The expression "complex contour" refers to a contour that comprises random technical curves which cannot be cut using the interpolation equipment of CNC WEDM machines. The two methods proposed approximate the arcs of random technical curves through arcs of a circle and can be implemented in practice on any WEDM machine. Even though they were devised for WEDM processing specifically, the methods proposed can also be used when dealing with the cutting of complex contours using other nonconventional means of processing, such as: LASER cutting, electron- or ion-beam cutting, plasma cutting, and even water jet cutting.

**KEYWORDS:** WEDM, plane complex contours, approximation of technical curves

## 1. CUTTING ALONG COMPLEX CONTOURS

Cutting complex contours using WEDM machining is one of the myriad possibilities of processing parts made from materials endowed with special properties, such as: great hardness, increased brittleness, low rigidity, or great geometrical complexity.

CNC WEDM machines are equipped with minicomputers which are capable of handling, in most cases, linear or circular interpolation functions which produce errors of 0.001 mm in the case of linear and circular interpolation of radii under 30 mm and of 0.002 mm in the case of circular interpolations of radii exceeding 30 mm [1].

When needing to cut certain complex contours which contain arcs of other plane technical curves, the problem posed is of approximating these by line segments or arcs of a circle, while ensuring the dimensional, shape, and positioning precision required by the technical prescriptions imposed on the processing.

Known methods of approximation of curves by line segments include: the chord method, the tangent method, the secant method, or the method of approximating curves by arcs of a parabola [2] or by using SPLINE curves [3].

When using approximation methods of curves by line segments, in order to achieve great precision, it was noticed that large-scale programs are needed. Consequently, when dealing with long contours that

require high precision, programs are likely to exceed the capabilities of the CN contouring commands. Some of these interpolation methods, such as the method of approximating curves by arcs of a parabola cannot be used due to the limited interpolation possibilities of the CN contouring facilities specific to certain types of machines.

## 2. METHODS OF APPROXIMATING PLANE TECHNICAL CURVES

In order to approximate plane technical curves, two methods are proposed for consideration: the osculating circle method and the three-point circle definition method. Both methods reveal cyclical structures which are illustrated by way of the logical diagrams shown in Figures 1 and 2 [4].

The osculating circle method (Figure 1) requires stating the initial conditions needed to make the approximation using the STACOND procedure. This procedure defines the pitch of the cycle ( $p$ ), the number of iterations to be carried out ( $i_{max}$ ), and the maximum acceptable value of the approximation error ( $e_{max}$ ).

The pitch of the cycle, which is usually expressed in angular form, and seldom in linear form, together with the number of iterations, are usually defined at initial values that take into account the precision imposed on the processing via the technical prescriptions formulated.

The values of these parameters can be altered if greater than admissible errors are experienced.

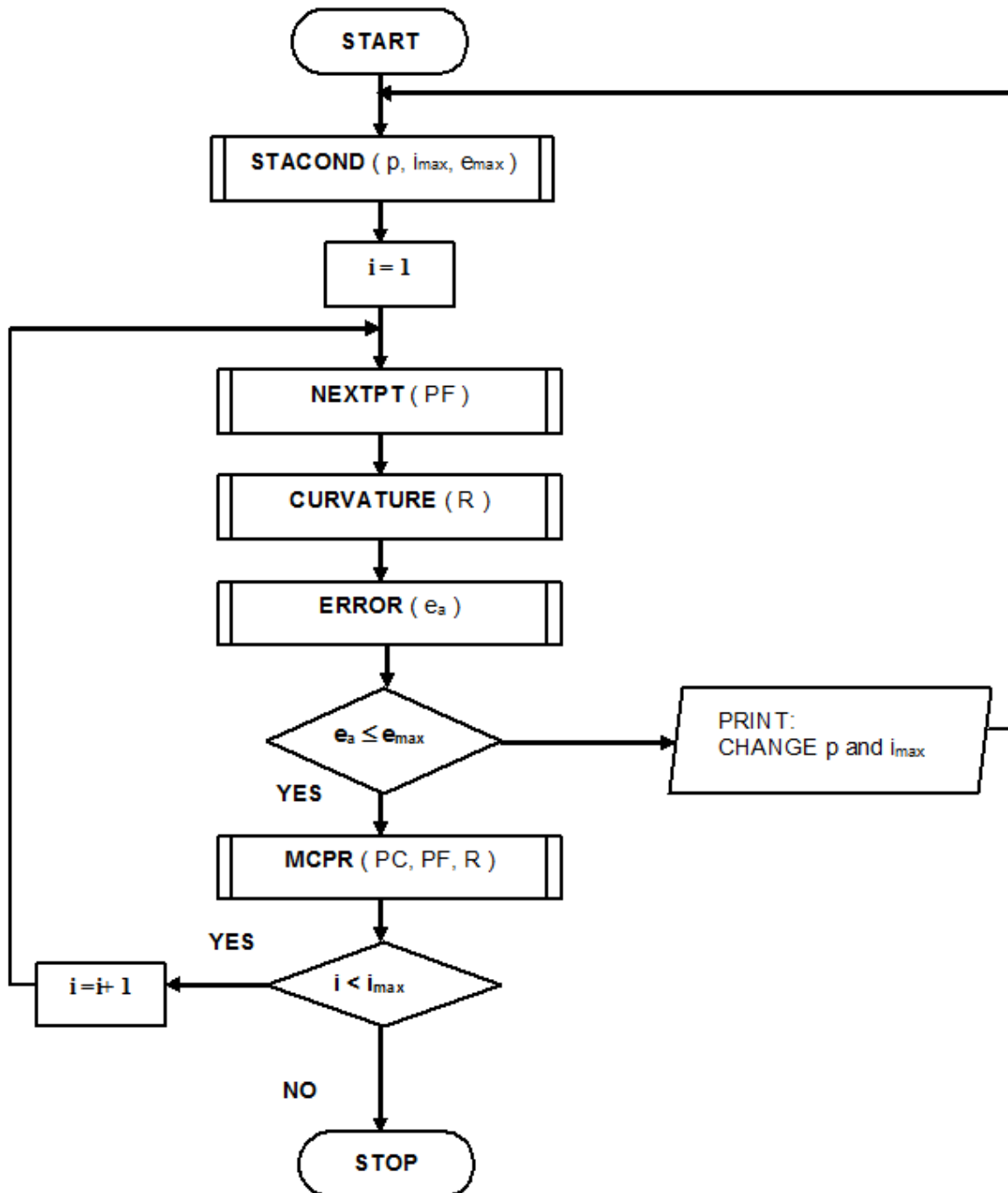


Figure 1. The osculating circle method - logical diagram

The maximum admissible approximation error ( $e_{\max}$ ) can be determined as a fraction of the tolerance imposed on the processing. The recommended value of this is  $e_{\max} = (0.1 \div 0.125) \cdot T_p$ , where  $T_p$  is the part's tolerance [4].

The completion of a step of the cycle means, from a geometrical perspective, the pinpointing of the final point (PF) of the step (Figure 1) using the NEXTPT procedure, followed by the calculation of the curvature radius (R) of the approximated curve at the midpoint of the step using the CURVATURE procedure.

The expressions of the curvature radii characterizing the most typical ways of representing plane curves are shown in Table 1 [4].

The ERROR procedure is used to calculate the approximation error ( $e_a$ ) of the step's midpoint and compare it to the admissible one ( $e_{\max}$ ). If  $e_a > e_{\max}$  the cycle's step (p) and number of iterations ( $i_{\max}$ ) are changed, thereafter resuming the cycle.

A motion is then generated following the trajectory of an arc of a circle whose radius is equal to the curvature radius of the approximated curve at the midpoint of the step (R), which joins the current point (PC) with the final point (PF) of the step (Figure 3).

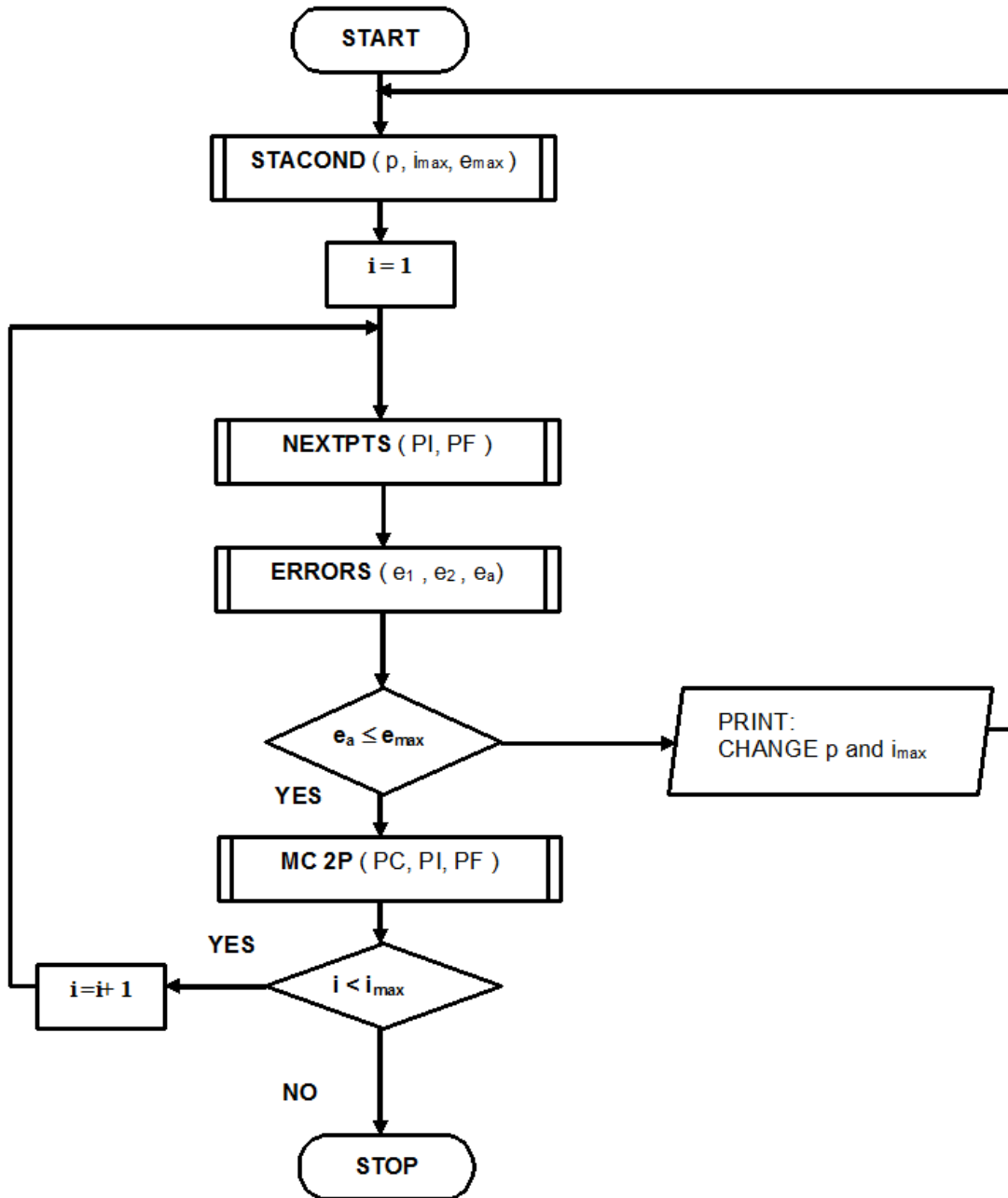


Figure 2. The three-point circle definition method - logical diagram

Table 1. Calculating the curvature radius based on the way of representing the approximated curve

No.	Way of representing the planar curve	The expression of the curvature of a curve at a point
1	<i>Explicit</i> $y = f(x)$	$R = \frac{(1 + y'^2)^{\frac{3}{2}}}{ y'' }$
2	<i>Parametric</i> $x = x(t)$ $y = y(t)$	$R = \frac{(x'^2 + y'^2)^{\frac{3}{2}}}{ x'y'' - x''y' }$
3	<i>Through a polar equation</i> $r = r(\theta)$	$R = \frac{(r'^2 + r^2)^{\frac{3}{2}}}{ 2 \cdot r'^2 + r^2 - r \cdot r'' }$

The arc of the circle used for the approximation will pass through points PC and PF and its radius will be equal to the curvature radius of the approximated curve which was calculated at the midpoint of the

step. The final point (PF) which results after completing one step of the cycle becomes the current point (PC) of the next step.

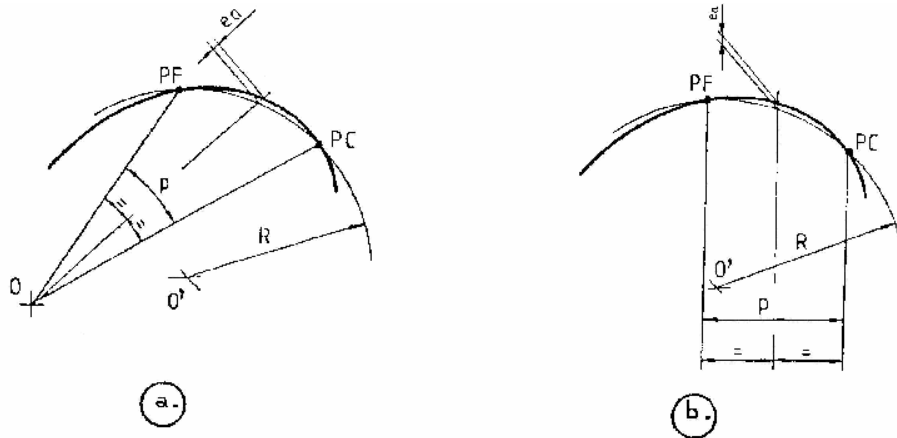


Figure 3. Approximating curves using the osculating circle method over one step of the cycle

The three-point circle definition method starts from the stage in which the required initial conditions of the approximation are laid out, in a manner similar to the one delineated for the osculating circle method.

Using the procedure called NEXTPTS we calculate two points on the approximated curve for each step, namely an intermediary point (PI) situated at the midpoint of the step, and the final point (PF) of the step, as shown in Figure 4.

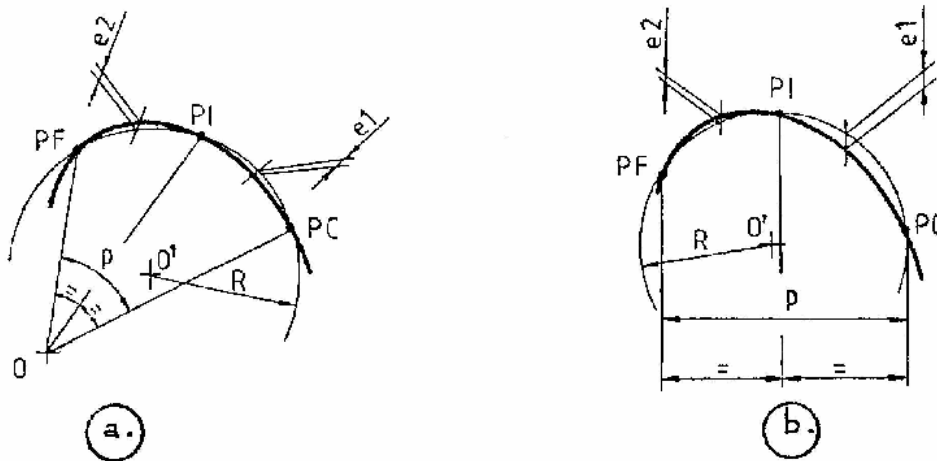


Figure 4. Approximating curves using the three-point circle definition method over one step of the cycle

Using the procedure called ERRORS we then calculate errors  $e_1$  and  $e_2$  that correspond to the two halves of the step (Figure 4), taking the greater value of the two to be the approximation error of the step. Put in other words,  $e_a = \max(e_1, e_2)$ . After checking the approximation error in a manner similar to the one used for the osculating circle method, we generate a movement on an arc of a circle defined by the current point (PC), the intermediary point (PI), and the final point (PF) of the step. Put another way, the arc of a circle used for the approximation will lie

on the circle defined by the three points: PC, PI, and PF.

### 3. PRACTICAL RESULTS

The two methods of approximating curves through arcs of a circle have been tested on two types of WEDM machines, according to the logical diagrams presented in Figures 1 and 2. The results obtained have led to the conclusion that these methods are vastly superior to the ones that use line segments to approximate curves, the precision of the processing being a lot better.

Figure 5 shows a sample complex contour which results at the intersection of four ellipses forming a cross pattern and having a common focal point. The processing program was written so as to approximate the arcs of an ellipse by arcs of a circle.



**Figure 5.** A complex contour processed using approximation

Approximating arcs of an ellipse by arcs of a circle was done using the osculating circle method presented in Figure 3a. Starting from the parametric equations of the ellipse, and using an angular step of  $15^\circ$  for each of the four ellipses, the contour requested was processed with a degree of accuracy of under 0.01 mm.

When cutting elliptical contours whose semi axes are smaller than 50 mm and whose angular step is no greater than  $15^\circ$ , the approximation errors are situated between 0.0001 – 0.005 mm [4]. The values of the approximation errors can be reduced all the way to a preset value by decreasing the step of the cycle; however, in order to ensure the correct functioning of the CN contouring equipment, it is recommended that the angular step be not less than  $1^\circ$ .

Even though they result in high precision, in the context of manufacturing planning time-saving considerations, these methods introduce applicability limitations when the maximum displayed value of the CN contouring equipment, which is usually  $\pm 999.999$  mm [4], has been exceeded. The same limitation applies in the case of the three-point circle definition method, when the steps of the cycle are extremely small.

To overcome the limitations of the two approximation methods discussed herein, the combined utilization of approximating curves through arcs of a circle and line segments is recommended. Therefore, in the case of areas of the curve where the curvature is less than 0.001 mm, line segment approximation methods can be used, whereas for curvatures in excess of 0.001 mm, methods of approximation through arcs of a circle can be used, these offering better approximation conditions [4].

Whereas the osculating circle method can only be used for the approximation of random technical curves that can be defined analytically (explicitly, parametrically, using polar coordinates, etc.), the three-point circle definition method can be used for the approximation of any type of plane curves.

#### 4. CONCLUSIONS

The approximation methods proposed can be applied when needing to cut any type of contour, regardless of its degree of complexity. Moreover, even though the methods were devised with WEDM processing in mind, they can be extended also for the cutting of plane complex contours using other nonconventional types of processing, such as LASER cutting, electron- or ion-beam cutting, plasma cutting, and even water jet cutting.

Whereas the osculating circle method can only be used for the approximation of random technical curves that can be defined analytically, the three-point circle definition method can be used for the approximation of any type of plane curves.

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